MONTHLY NOTICES

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Rev. T. E. R. PHILLIPS, PRESIDENT, in the Chair.

Ronald Allan, 204A High Holborn, London, W.C. 1 ;
Carlyle S. Beals, M.A., Ph.D., Assistant Professor of Physics, Accadia University, Wolfville, Nova Scotia, Canada ;
Henry Whitworth Davis, M.I.C.E., M.I.M.E., The Manor House, Earsdon, Northumberland ;
Captain Thomas Leonard Gillett, Elmdon, Adelaide Grove, East Cowes, Isle of Wight ;
Rudolf Kingslake, M.Sc., A.R.C.S., D.I.C., Beit Scientific Research Fellow, Imperial College of Science and Technology, S. Kensington ; 171 Grove Lane, Camberwell, London, S.E. 5 ;
P. J. van Rhijn, Director of the Kapteyn Astronomical Laboratory, Groningen, Holland ;
Theodore Thomas Ambrose Varley, Feizor, Jersey Road, Heston, Middlesex ; and
Rev. Frederick James Wright, F.R.M.S., 332 Audley Range, Blackburn, Lancashire,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Lyndon Bolton, M.A., 4 Shakespeare Road, Bedford (proposed by E. A. Milne) ;
R. H. Curtiss, Director of the Detroit Observatory, Ann Arbor, Michigan, U.S.A. (proposed by M. L. Zimmer) ;

Wendell Phillips Hoge, Mount Wilson Observatory, Pasadena, Cal., U.S.A. (proposed by E. Hubble); and
William Johnson, 4 Lynwood Road, London, S.W. 17 (proposed by F. Robbins).

One hundred and twenty-eight presents were announced as having been received since the last meeting, including, amongst others :—

Dreyer (J. L. E.): Catalogue of Manuscripts at Oxford and at Cambridge on Planetary Theory and Planetary Tables (MS.).
Gifford (J. W.): Lens Computing by Trigonometrical Trace.
Houtman (Frederick de): Star Catalogue, 1603 (presented by Prof. Turner and Mr. Knobel).
Lembang, Bosscha-Sternwarte: Annalen, vol. 1.

The President presented the Gold Medal to Professor Frank Schlesinger.

Professor Schlesinger then delivered the first George Darwin Lecture, taking as his subject, "Astronomical Photography of Precision."

(It is hoped to publish the lecture in a later number of *Monthly Notices*.)

An Emission Line in the Solar Spectrum near K.

By J. Evershed, F.R.S.

Photographs of the spectrum of the limb of the sun obtained recently reveal the presence near K of a rather faint narrow emission line, which does not appear to have been described previously. Unlike the emission lines of calcium, H_2 and K_2, this radiation appears to be uniformly distributed, and I believe is always present. It is, in fact, the only emission line in the visible region of the spectrum which has this character. It varies in apparent intensity, being clearly seen only in photographs obtained in a non-diffusive sky.

In spectra obtained with the slit radial to the limb, the line can be traced inwards over the disc for about a minute of arc, but it seems to be entirely absent in the central parts of the disc and in spot regions except when these are close to the limb.

The approximate position of the line is 3934·80 I.U., or in Rowland's system of solar wave-lengths 3934·94. In Rowland's Preliminary Table seven absorption lines are given between the line 3932·785 and the line 3935·463; that is to say, superposed upon the broad absorption bands of the K line, and one of these, 3934·818, of intensity 0, is near to the new line. I have studied a great number of high-dispersion

plates of this region, representing the centre of the disc and the limb, but have failed to detect this absorption line, or any of the others, although two of them are entered in the table as of intensity 8. There are similar discrepancies in the neighbourhood of H, where Rowland gives two lines of intensity 6, of which no trace can be seen on any of my plates, whether grating or prism spectra.

The plates which show the emission line most clearly form part of a series I am photographing at Ewhurst for the purpose of measuring the rotation speed of the prominences at a considerable height above the chromosphere. The spectra represent sections of prominences appearing on the eastern or western limbs, and the slit, which is tangent to the limb, in some cases intersects a small part of the disc. Such spectra usually show the emission line superposed on the shading of K. It is also seen in some spectra taken with the slit radial to the sun and intersecting the limb in a higher latitude than is represented by the prominence spectra.

As to the origin of the line, it might be suggested that it was the K line of calcium much displaced by motion, and having some analogy with the " stationary " calcium lines in early-type stars. In this case a similar line would appear on the red side of H, but it does not appear to be present. The displacement with regard to the normal solar position of K would imply a recession of 85 km./sec., so that if it were the K line, due to external very tenuous calcium, it would be necessary to suppose that the solar system was moving through the calcium cloud with a velocity at least as great as this. The suggestion is, however, negatived by the fact that my measures clearly show that the line is displaced normally to red and to violet on the west and east limbs by reason of the solar rotation.

I give below the values obtained from six plates, which are the only ones I have obtained that give measurable images of the line.

Date.	Latitude East.	λ I.U.	Date.	Latitude West.	λ I.U.
1926 Aug. 11	+14°	3934·783	1926 Aug. 7	−13°	3934·822
1926 Aug. 14	+15°	3934·797	1926 Aug. 11	−14°	3934·824
1927 Jan. 23	− 8°	3934·773	1927 Jan. 23	+ 8°	3934·835
Means East	12°	3934·784	Means West	12°	3934·827

Mean of East and West 3934·806 ± ·002.
Observed Rotation Shift 0·022 A.

Applying the usual corrections to the observed shift, the equivalent value of the sidereal rotation is 1·86 km./sec., which agrees almost exactly with the normal value of the reversing layer in latitude 12°.

It is to be noted that the measures have been referred to the comparison spectrum, impressed on all the plates, of the iron arc, and not to the solar absorption lines, for the reason that recent measures of the wave-lengths of the iron lines by St. John and others (*Transactions of the International Astronomical Union*, **1**, 41) are far more reliable than Rowland's values of the solar lines. But in comparing a terrestrial

source with the sun, we have to reckon with Einstein's predicted shift towards red in the sun. The mean value of the wave-length obtained above is therefore subject to a correction of − 0·008 A to reduce it to the terrestrial equivalent. If the origin of this emission line should be found on earth, it would therefore have the value 3934·798 I.U.

It is of interest in this connection to mention that in carrying out the original intention of measuring the rotation velocities of the prominences I have found that the H and K lines at a height of between 10″ and 30″ above the chromosphere are each of them displaced towards red, when referred to the terrestrial iron lines. The mean displacement derived from twenty-nine prominences is the same for H and K, and amounts to + 0·009A, the predicted value being + 0·008A.

The results of the prominence measures will be the subject of a separate communication, when a sufficient number of plates has been studied.

Ewhurst:
1927 *March* 6.

The Relative Effective Temperatures of Twenty-two Stars of Early Type.

By W. M. H. Greaves, C. Davidson, and E. Martin. (Plate 6.)

(*Communicated by the Astronomer Royal.*)

Introduction.—If the temperature of a "black" body α be known, and if another black body β can be compared with it by measuring the relative intensities of the light emitted by α and β, such a comparison being made for each of two different wave-lengths, then the temperature of β can be deduced by the use of Planck's formula. Temperatures of stars determined in this way are called effective temperatures, and it must be borne in mind that the determination of effective temperature is simply a determination of colour; that is to say, a determination of the relative amounts of red and blue light present in the continuous spectrum. The conversion into temperature by means of Planck's formula introduces the assumption that the stars are full radiators, but it is generally believed that effective temperatures determined in this way represent the photospheric temperatures of the stars fairly well.

In a previous paper * an account has been given of a method by means of which a comparison between two light sources α and β can be effected. The following is a brief summary: The spectra of α and β are photographed on the same plate and the densities of the photographic deposit at different wave-lengths measured in a microphotometer. To convert the difference of densities between the two spectra into measures of relative light intensity a scale on the plate is required by means of which measures of density can be converted into measures of light intensity. This is effected by placing a coarse wire grating in the optical train with its dispersion at right angles to the dispersion of

* *M.N.*, 86, 1925 November.

the prism. The central spectra of both α and β are then flanked by lateral spectra, and the ratio of the light falling into the central spectrum to that falling into each of the first order lateral spectra is a constant which depends only on the dimensions of the grating, and is independent of wave-length. The difference of density between the central and lateral spectra can be measured at any wave-length, and, the corresponding light ratio being known, a scale is provided for converting density differences into light ratios. Actually one works, not with the light ratios themselves, but with their logarithms multiplied by 0·4, that is, in terms of stellar magnitudes.

It is shown in the previous paper that if for any wave-length λ, D_1, and D_2 be the measured densities of the central and first order lateral spectra of the light source α, and if D_3 and D_4 be the corresponding measures for β, then the magnitude difference Δm_λ between the two spectra is given by the formula

$$\Delta m_\lambda = \frac{(D_1 + D_2) - (D_3 + D_4)}{(D_1 - D_2) + (D_3 - D_4)} \times g \quad . \quad . \quad (1)$$

where g is the magnitude difference between the central and lateral spectra in each case and is simply the "grating constant."

The difference of gradient $G_1 - G_2$ between the spectra is defined as the slope of Δm_λ with regard to reciprocal of wave-length multiplied by a certain constant, the formula for $G_1 - G_2$ being

$$G_1 - G_2 = \frac{d\Delta m_\lambda}{d(1/\lambda)} \times 0{\cdot}922 \quad . \quad . \quad . \quad (2)$$

λ being expressed in microns.

It is shown in the previous paper that if T_1 and T_2 be the effective temperatures of α and β, then T_1 and T_2 are connected by the relation

$$\frac{c_2}{T_2} - \frac{c_2}{T_1} = G_1 - G_2 + \left[\frac{c_2}{T_1} \frac{1}{e^{x_1} - 1} - \frac{c_2}{T_2} \frac{1}{e^{x_2} - 1} \right] \quad . \quad (3)$$

where

$$x_1 = c_2/\lambda T_1, \qquad x_2 = c_2/\lambda T_2, \qquad \text{and} \qquad c_2 = 14{,}320.$$

In practice, $G_1 - G_2$ is derived from a series of measures extending over a considerable range of wave-length, and the mean value of λ in this range is substituted for λ in formula (3).

The results here presented consist of determinations of the gradient differences between pairs of stars of spectral types A and B with the object of determining the relative gradients of all the stars on the observing list. This done, then, if the temperature of any one star be known, those of all the others can be deduced. Actually the gradients have been converted into effective temperatures on two assumptions : *firstly*, that the mean of certain Ao stars corresponds to a temperature of 10,000° K ; *secondly*, that the same mean corresponds to a temperature of 13,000° K.

Fig. 1 is a diagram showing the observations that have been made. Stars of type B are arranged at the top of the diagram and stars of type

A at the bottom. A line joining two stars on the diagram means that a series of comparisons have been made between the two stars in question.

Each determination of gradient difference has to be corrected for the effect of selective atmospheric absorption. If the stars α and β be photographed at zenith distances z_1 and z_2, and if the observed difference of gradient be ΔG, then the true gradient difference $\Delta G'$ is given by

$$\Delta G' = \Delta G + A(\sec z_1 - \sec z_2) \quad . \quad . \quad (4)$$

where A is the constant of selective atmospheric absorption. Each pair of stars was observed at altitudes as nearly equal as possible, so as to make the coefficient of A small. The constant A was determined by obtaining series of observations of two pairs of stars, viz. α Lyræ and ϵ Ursæ Majoris and α Lyræ and ζ^1 Ursæ Majoris. These observations were made at different altitudes, so that a wide range of values of $\sec z_1 - \sec z_2$ was obtained in each case. On being reduced they give A

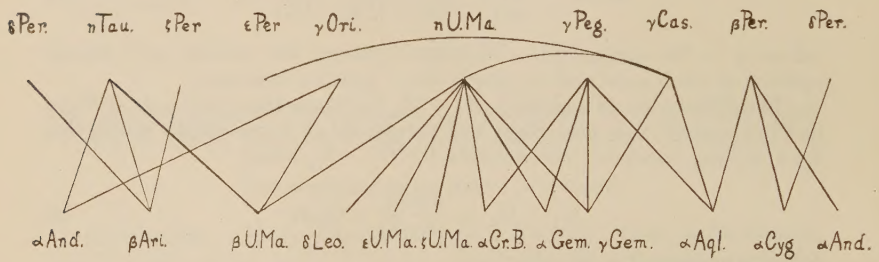
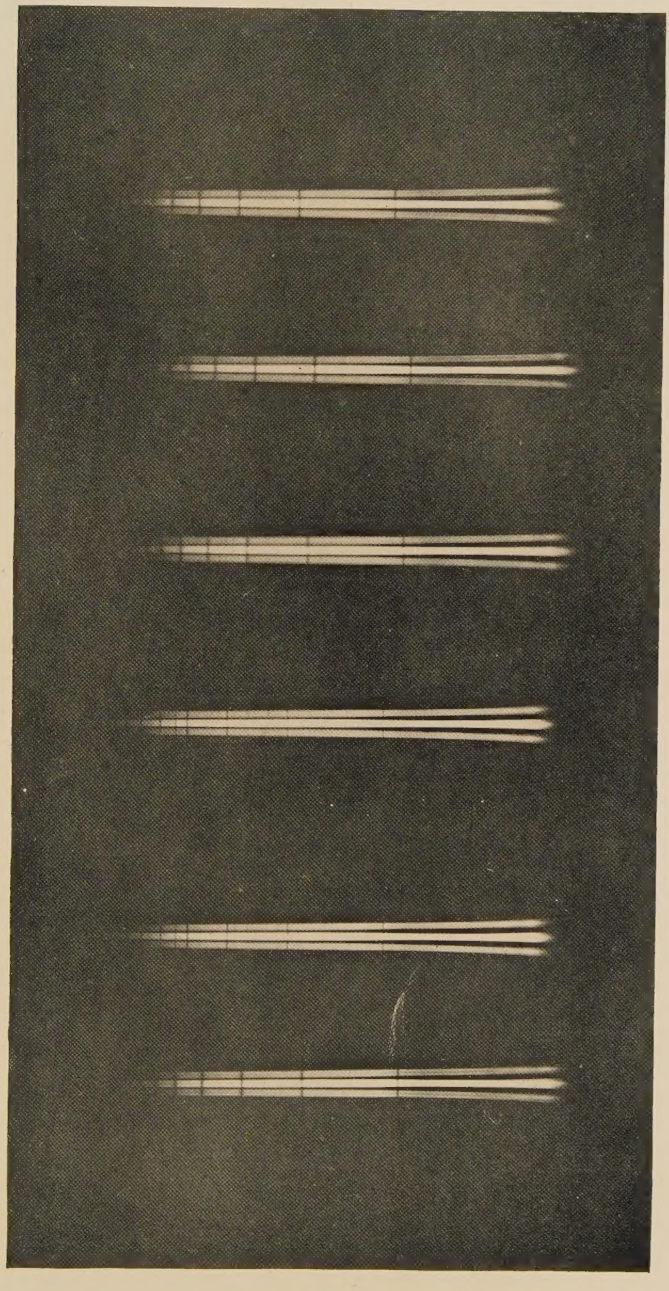


Fig. 1.

and also the true gradient difference $\Delta G'$ for each pair. The details are given below. The value of A so determined was applied to all the observations.

In the case of each comparison, the exposures were chosen so as to make the mean densities of the two spectra as nearly equal as possible. Owing to the fact that in no case have stars differing widely in spectral type been compared, the gradient differences determined are comparatively small, and the spectra are fairly well matched for all wavelengths, the difference never exceeding that between the central and lateral spectra of the first order.

The spectra were photographed with the 30-inch Thompson reflector. A description of the optical train and also of the microphometer used for measuring the plates was given in the previous paper. At the commencement of the work, Marion Iso-Record plates were used. These plates are sensitive to the yellow, but have a region of low sensitivity in the green. Measurements were made at 8 points in the blue, extending from λ 3994 to λ 4204; at 4 points on the blue side of the minimum in the green, extending from λ 4952 to λ 5073; and at 4 points in the yellow, extending from λ 5454 to λ 5624. At a later stage Ilford Panchromatic plates giving a more even sensitivity curve



Spectra taken with Prism and Grating.

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and also a longer base line were substituted. The red now appears on the plates, but is out of focus (the camera lens being photographically corrected), and the measurements were made, as before, in the blue, but at 8 points in the yellow, extending from λ 5385 to λ 5789, good focus having been secured for these two regions. Each pair was measured by two observers, the first measuring the central and one lateral spectrum of each star and the second the central and the other lateral spectrum. The plate reproduced is a positive of one of the photographs taken with the Panchromatic plates. Observations were confined to nights on which, as far as can be judged, the sky was uniformly transparent.

Determination of the Grating Constant g.—The grating used was made by Messrs. Grubb, Parsons & Co., and was received from the makers on 1925 November 14. It consists of wires of non-corrodible material approximately 1·5 mm. in diameter and 1·5 mm. apart. The relative sizes of wire and space were measured by a micrometer in 23 different regions of the grating, measures of ten wires and of ten spaces being taken in each region. From these samples the value of mean space ÷ (mean wire + mean space) came out to be 0·4824. Denoting this by x, the value of the grating constant g was calculated from the formula

$$0{\cdot}4g = \log [\pi^2 x^2 / \sin^2 (\pi x)].$$

With the above value of x this gives $g = 0{\cdot}895$ stellar magnitudes.

As a check on the method it was thought desirable to obtain a value for g by direct observation of the same kind as the star observations. For this purpose a half-watt incandescent lamp was installed on the roof of a building some 600 feet away and the instrument set and focussed on it. The details are the same as those given in the previous paper. Two semicircular stops were constructed, each of which covered half the aperture of the 30-inch reflector, and exposures were made on the lamp (1) with a full aperture, (2) with the top half of the aperture, and (3) with the bottom half, the times of exposure being the same in each case. The plates were measured and reduced with $g = 0{\cdot}895$, the magnitude differences Δm_1, Δm_2 between the full and the top half, and the full and the bottom half being deduced from equation (1). If I, I_1, I_2 denote the light intensities of the full beam and the two half beams respectively,

$$0{\cdot}4\Delta m_1 = \log (I/I_1)$$
$$0{\cdot}4\Delta m_2 = \log (I/I_2).$$

Put

$$0{\cdot}4\Delta m = \log [I/\tfrac{1}{2}(I_1 + I_2)].$$

Δm can then be calculated from the measured values of Δm_1 and Δm_2. But $I_1 + I_2 = I$, and so Δm should equal $0^m{\cdot}75$ magnitudes. The closeness of the agreement between the observed value of Δm with the theoretical one $0^m{\cdot}75$ should furnish a test of the accuracy of the method.

Five plates were taken for this purpose, and each plate was measured at a number of different wave-lengths. The results are shown in Table I. For each plate the measured values of Δm are given in order of increasing wave-length. The wave-lengths vary from λ 4100 to λ 5600.

TABLE I.

Observed Values of Δm.

	Plate I. Δm.	II. Δm.	III. Δm.	IV. Δm.	V. Δm.
	0·75	0·75	0·68	0·75	0·78
	0·70	0·73	0·73	0·75	0·78
	0·70	0·75	0·73	0·73	0·73
	0·75	0·75	0·70	0·73	0·73
	0·75	0·73	0·73	0·78	0·75
	0·78	0·75	0·75	0·73	0·75
	0·79	0·78	0·78	0·75	0·78
	0·73		0·75	0·78	
Means	0·74	0·75	0·73	0·75	0·76

The mean of the results for five plates is 0·75 magnitudes. Plates I., II. and III. were Ilford Panchromatic plates and Plates IV. and V. were Marion Iso-Record plates.

The agreement between the observed and computed results is satisfactory, and the observations have been reduced with $g = 0{\cdot}895$ accordingly.

Atmospheric Absorption.—For the determination of the constant A of selective atmospheric absorption a series of comparisons were made between α Lyræ and ϵ Ursæ Majoris and α Lyræ and ζ^1 Ursæ Majoris. The results are shown in Table II., which gives the measured gradient difference ΔG in the sense α Lyræ − ϵ or ζ^1 Ursæ Majoris and the quantity $\sec z_1 - \sec z_2$ in each case. Here z_1 is the zenith distance of α Lyræ at the time of observation and z_2 the zenith distance of the other star.

TABLE II.

Observations for Determination of A.

α Lyræ − ϵ Ursæ Majoris.		α Lyræ − ζ^1 Ursæ Majoris.	
ΔG.	$\sec z_1 - \sec z_2$.	ΔG.	$\sec z_1 - \sec z_2$.
− 0·13	+ 0·36	− 0·46	+ 0·74
− 0·63	+ 0·74	− 0·06	+ 0·31
− 0·87	+ 0·80	− 0·59	+ 0·80
− 0·84	+ 0·82	− 0·11	+ 0·36
+ 0·30	− 0·32	− 0·08	+ 0·40
+ 0·10	− 0·31	+ 0·50	− 0·33
+ 0·20	− 0·26	+ 0·20	− 0·34
+ 0·26	− 0·22	+ 0·26	− 0·22
+ 0·54	− 0·94	+ 0·38	− 0·22
+ 0·43	− 0·41	+ 0·41	− 0·79
+ 0·53	− 0·71	+ 0·52	− 0·52
+ 0·34	− 0·58	+ 0·35	− 0·68
..	..	+ 0·27	− 0·52

Forming in each case the equation of condition

$$\Delta G' = \Delta G + A(\sec z_1 - \sec z_2)$$

least square solutions give the following results for the constant A and the true gradient differences $\Delta G'$ between α Lyræ and the Ursa Major stars.

From Comparisons of α Lyræ and ϵ Ursæ Majoris.

$$A = 0{\cdot}82 \pm 0{\cdot}05$$
$$\Delta G' = -0{\cdot}05 \pm 0{\cdot}03$$

From Comparisons of α Lyræ and ζ^1 Ursæ Majoris.

$$A = 0{\cdot}62 \pm 0{\cdot}06$$
$$\Delta G' = +0{\cdot}07 \pm 0{\cdot}03$$

A combined solution of the two series gave

$$A = 0{\cdot}73 \pm 0{\cdot}05$$
$$\Delta G_1 = -0{\cdot}04 \pm 0{\cdot}04$$
$$\Delta G_2 = +0{\cdot}07 \pm 0{\cdot}04$$

where ΔG_1 and ΔG_2 denote the gradient differences α Lyræ $-$ ϵ Ursæ Majoris and α Lyræ $-$ ζ^1 Ursæ Majoris. The observations were reduced with A = 0·73.

Results.—Table III. gives for each comparison between two stars the observed gradient as deduced from the measures, the correction for atmospheric absorption, and the deduced gradient corrected for atmosphere. Thus in the case of the first comparison between γ Orionis and β Ursæ Majoris the observed gradient is + 0·49, the atmospheric correction is − ·07, the corrected gradient being + 0·42. The mean of each group of comparisons is also given as well as the spectral types of the stars compared as given in the Henry Draper Catalogue.

Table III.

γ Peg. − γ Gem. B2. Ao.			γ Ori. − β U.Ma. B2. Ao.		
+·12	+·34 = +·46		+·49	−·07 = +·42	
+·38	−·01 = +·37	+·40	+·03	−·09 = −·06	+·22
+·32	−·06 = +·26		+·48	−·12 = +·36	
+·55	−·06 = +·49		+·25	−·09 = +·16	
η U.Ma. − γ Gem. B3. Ao.			η U.Ma. − α Gem. B3. Ao.		
+·17	+·01 = +·18		+·24	+·07 = +·31	
+·33	+·05 = +·38	+·24	+·38	+·09 = +·47	+·33
+·37	−·04 = +·33		+·46	−·06 = +·40	
+·06	−·01 = +·05		+·24	−·10 = +·14	
η U.Ma. − δ Leo. B3. A3.			η U.Ma. − ϵ U.Ma. B3. Aop.		
+·58	−·08 = +·50		+·32	−·01 = +·31	
+·72	−·06 = +·66	+·58	+·24	−·01 = +·23	
+·61	−·06 = +·55		+·16	−·01 = +·15	+·21
+·67	−·06 = +·61		+·33	−·01 = +·32	
			+·07	−·01 = +·06	

TABLE III.—*continued.*

η U.Ma. — β U.Ma. B3. Ao.		η U.Ma. — ζ^1 U.Ma. B3. A2*p*.	
+·14 −·04 = +·10	+·14	+·34 ·00 = +·34	+·39
+·13 −·01 = +·12		+·42 +·01 = +·43	
+·12 −·01 = +·11		+·36 ·00 = +·36	
+·25 −·01 = +·24		+·47 ·00 = +·47	
		+·35 ·00 = +·35	

α Vir. — α Lyr. B2. Ao.		η U.Ma. — α Cr.B. B3. Ao.	
+·11 +·28 = +·39	+·43	+·33 −·04 = +·29	+·29
+·23 +·24 = +·47		+·39 −·04 = +·35	
		+·48 −·04 = +·44	
		+·13 −·04 = +·09	

γ Cas. — α Aql. Bo*p*. A5.		γ Peg. — α Cr.B. B2. Ao.	
+·51 ·00 = +·51	+·45	+·23 −·26 = −·03	+·08
+·43 ·00 = +·43		−·03 +·03 = ·00	
+·37 +·04 = +·41		+·05 +·07 = +·12	
		+·38 −·01 = +·37	
		+·07 −·10 = −·03	
		+·13 −·07 = +·06	

γ Cas. — α Cyg. Bo*p*. A2*p*.		γ Peg. — α Aql. B2. A5.	
+·24 +·05 = +·29	+·16	+·57 +·03 = +·60	+·72
+·02 +·04 = +·06		+·89 +·04 = +·93	
+·14 +·01 = +·15		+·63 −·03 = +·60	
+·20 ·00 = +·20		+·71 ·00 = +·71	
+·08 −·01 = +·07		+·75 ·00 = +·75	
+·22 −·02 = +·20			

β Per. — α Aql. B8. A5.		β Per. — α Cyg. B8. A2*p*.	
+·40 −·18 = +·22	+·36	+·09 +·05 = +·14	+·31
+·37 +·01 = +·38		+·32 +·06 = +·38	
+·52 +·04 = +·56		+·33 −·02 = +·31	
+·51 −·02 = +·49		+·36 +·04 = +·40	
+·17 −·01 = +·16		+·28 +·04 = +·32	

δ Per. — α Cyg. B5. A2*p*.		β Per. — α And. B8. Ao*p*.	
+·09 −·04 = +·05	+·20	−·01 ·00 = −·01	−·02
+·37 ·00 = +·37		−·02 +·01 = −·01	
+·25 −·07 = +·18		−·06 −·03 = −·09	
+·20 −·01 = +·19		+·06 −·02 = +·04	

TABLE III.—*continued.*

γ Cas. — ε Per. Bo*p*. B1.		
−·23 − ·04 = −·27		
−·34 − ·04 = −·38	−·34	
−·55 + ·05 = −·50		
−·18 − ·04 = −·22		
η Tau. — α And. B5*p*. Ao*p*.		
−·01 − ·07 = −·08		
−·17 − ·06 = −·23	−·18	
−·09 − ·07 = −·16		
−·23 − ·04 = −·27		
δ Per. — β Ari. B5. A5.		
+·60 − ·01 = +·59		
+·60 − ·07 = +·53	+·50	
+·37 − ·01 = +·36		
+·52 − ·01 = +·51		
ζ Per. — β Ari. B1. A5.		
+·06 − ·03 = +·03		
+·05 − ·01 = +·04	·00	
−·01 − ·01 = −·02		
−·06 − ·01 = −·07		
η Tau. — β Ari. B5*p*. A5.		
+·37 − ·01 = +·36		
+·52 − ·02 = +·50	+·38	
+·36 − ·07 = +·29		
γ Cas. — γ Gem. Bo*p*. Ao.		
−·01 + ·02 = +·01		
−·03 + ·04 = +·01	+·04	
+·14 − ·03 = +·11		
+·01 + ·01 = +·02		
η Tau. — β U.Ma. B5*p*. Ao.		
+·07 + ·04 = +·11		
−·01 + ·09 = +·08	+·07	
+·01 + ·06 = +·07		
−·02 + ·02 = ·00		
γ Cas. — η U.Ma. Bo*p*. B3.		
−·05 − ·18 = −·23		
−·18 − ·12 = −·30	−·24	
−·03 − ·16 = −·19		
γ Ori. — α And. B2. Ao*p*.		
−·14 − ·01 = −·15		
+·28 − ·18 = +·10	−·01	
+·14 − ·09 = +·05		
+·19 − ·24 = −·05		
α Lyr. — ε U.Ma. Ao. Ao*p*.		
(From previous section)	−·04	
α Lyr. — ζ¹ U.Ma. Ao. A2*p*.		
(From previous section)	+·07	

In completing the reduction, the means of all the groups in Table III. were given equal weight, and by a method of least squares the gradients of all the stars relative to η Ursæ Majoris (as standard) were obtained. The gradients were then all increased by + 0·25 to make zero the mean of all the stars on the list (five in number) of Harvard type Ao (excluding the Ao*p*'s). The resulting gradients are given in the seventh column of Table IV., which also gives for each star, the Harvard spectral type, the apparent visual magnitude, the parallax (from Schlesinger's catalogue), the colour index, and also the absolute magnitude whenever the parallax is greater than 0″·020. The colour indices are those given by King.*

From these gradients, effective temperatures (by formula (3) above) can be determined, provided the effective temperature corresponding

* *H.A.*, **76**, No. 6.

to zero gradient (that is, to mean Ao) be known. In the eighth and ninth columns of Table IV. temperatures on two bases are given, that in the eighth column on the assumption that zero gradient corresponds to a temperature of 10,000° K, and that in the ninth on the assumption that zero gradient corresponds to a temperature of 13,000° K.

The last column gives for purposes of comparison the gradients determined by Sampson.* The figures actually are Sampson's gradients † relative to Polaris, corrected for atmospheric absorption by the formula given in Sampson's paper, and diminished in each case by 1·32, to make the mean of Sampson's gradients identical with the mean gradient of the same stars according to the present determinations.

TABLE IV.

Results Referred to Mean Ao.

Star.	Type H.D.	App. Vis. *m*.	π.	Abs. M.	Colour Index.	G.	T. Ao= 10,000°.	T. Ao= 13,000°.	Gradient (Sampson).
γ Cas.	B0*p*	2·3	+·042±9	+0·4	−0·26	0·00	10,000	13,000	+·55
ζ Per.	B1	2·9	+·003±11	..	+0·16	−0·40	7,600	9,100	..
ε Per.	B1	3·0	−·006±11	..	−0·24	+0·34	13,800	22,000	−·05
γ Peg.	B2	2·9	..	..	−0·20	+0·27	12,800	19,100	+·29
γ Ori.	B2	1·7	+·025±7	−1·1	−0·33	+0·19	11,800	16,700	+·32
α Vir.	B2	1·2	+·015±10	−2·5	−0·49	+0·39	14,700	24,300	..
η U.Ma.	B3	1·9	+·002±11	..	−0·18	+0·25	12,500	18,400	+·32
δ Per.	B5	3·1	+·011±11	..	−0·13	+0·05	10,400	13,900	..
η Tau.	B5*p*	3·0	+·013±10	..	−0·08	+0·02	10,100	13,400	−·26
β Per.	B8	2·1 ‡	+·033±10	−0·3	..	+0·08	10,700	14,300	−·13
α And.	A0*p*	2·2	..	..	−0·08	+0·17	11,500	16,100	−·13
γ Gem.	A0	1·9	+·049±6	+0·4	+0·09	−0·05	9,600	12,400	..
α Gem.	A0	2·0	+·083±4	+1·6	+0·10	+0·01	10,100	13,200	..
β U.Ma.	A0	2·4	+·053±2 §	+1·0	−0·04	+0·01	10,100	13,200	+·17
ε U.Ma.	A0*p*	1·7	+·045±5 §	0·0	+0·16	+0·02	10,100	13,400	+·07
α Cr.B.	A0	2·3	+·059±10 +·044±2 §	+0·9	−0·08	+0·07	10,600	14,200	..
α Lyr.	A0	0·1	+·130±4	+0·7	−0·05	−0·04	9,700	12,600	−·21
ζ^1 U.Ma.	A2*p*	2·4	+·044±5 +·048±2 §	+0·7	+0·03	−0·13	9,100	11,500	+·05
α Cyg.	A2*p*	1·3	+·011±8	..	+0·09	−0·19	8,700	10,900	−·12
δ Leo.	A3	2·6	+·084±10	+2·2	+0·11	−0·33	8,000	9,700	..
β Ari.	A5	2·7	+·070±9	+1·9	+0·13	−0·40	7,600	9,100	−·37
α Aql.	A5	0·9	+·210±6	+2·5	+0·23	−0·40	7,600	9,100	..

* *M.N.*, **85**, 1925 January.

† Reversed in sign. The gradients as defined here are opposite in sign to Sampson's.

‡ Maximum. § Cluster parallaxes.

From a discussion of the residuals, the probable error of a single observation of gradient difference between two stars is ± 0·09. The probable errors of the final gradients in Table IV. are about ± 0·04.

It is clear from a casual inspection of the sixth and seventh columns of Table IV. that there is a strong correlation between the Greenwich

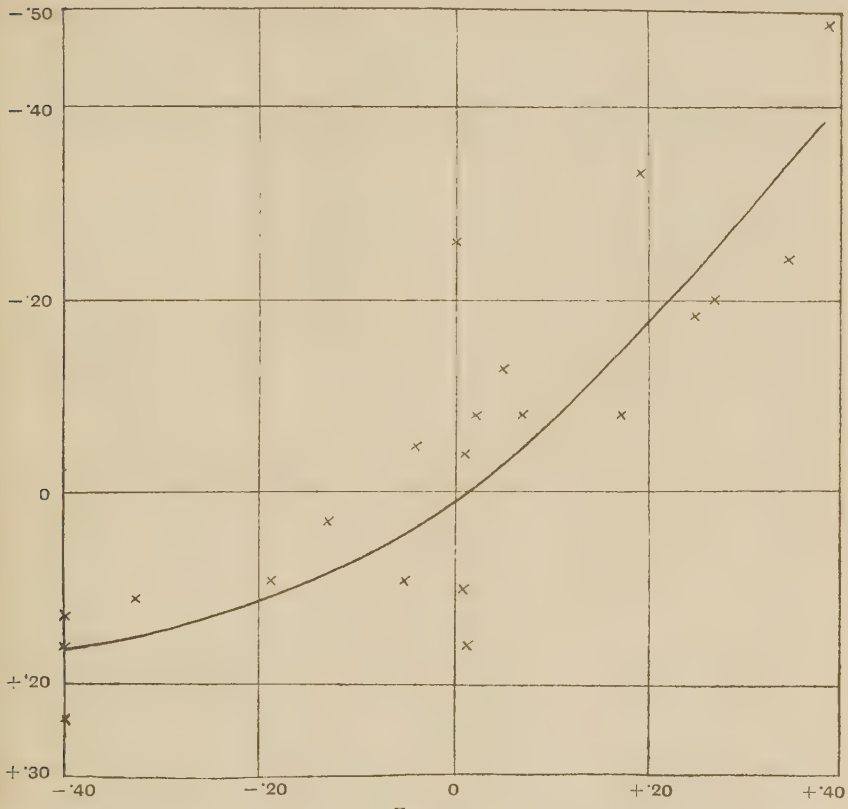

Fig. 2.

Abscissa = Gradient.
Ordinate = Colour Index (King).

gradients and the Harvard colour indices. Fig. 2 shows this correlation. The abscissæ are gradients, and the ordinates are colour indices.

It seems fairly evident that the correlation is expressed by a curved and not by a straight line. The gradients and the colour indices are both measures of colour, but the gradients are derived from measures of the continuous spectrum between the absorption lines, whilst the visual and photographic magnitudes from which the colour indices are derived are the results of measures in integrated light. In the observation of the photographic magnitudes all the absorption lines

of the Balmer Series from H_β onwards are integrated; and it is to be expected that at type Ao, where the Balmer Series is at its maximum strength, the colour indices would be greater than if they had been obtained from measures in the continuous spectrum only. In other words, it was to be expected that the diagram in fig. 2 would be curved and convex towards the axis of gradients, as is the case.

In connection with this remark, γ Cassiopeiæ is of special interest. This star possesses a peculiar spectrum, the absorption lines of the Balmer Series being replaced by emission lines. The replacing of absorption by emission in the blue would decrease the colour index, and γ Cassiopeiæ ought therefore to lie above the correlation curve, which in point of fact it does. In drawing the smooth curve on the diagram, γ Cassiopeiæ has been ignored.

It will be noticed that the temperatures based on mean Ao = 10,000° K are in conflict with those obtained by Fowler and Milne * from considerations of line intensity. Fowler and Milne deduce a formula connecting the temperature at which a given series of lines reaches maximum intensity, with the partial electron pressure P_e in the reversing layer and with certain atomic constants (ionisation potentials, etc.). Assuming that the temperature of stars of type Ao, for which the Balmer Series reaches maximum strength, to be 10,000° K, they obtain $P_e = 1{\cdot}3 \times 10^{-4}$ atmospheres, and using this value they obtain for B2 stars, for which the helium series He I attains maximum strength, T = 16,000° K. The discordance is manifest. A portion of it seems to be due to the fact that there are a very limited number of stars on our list, and actually the redder (*i.e.* the cooler) of the early B-type stars have been accidentally selected. King gives for the mean colour index of stars of type B2 the value $-0^{m}{\cdot}34$. According to the curve in fig. 2 this would correspond to a gradient of about + 0·36, and for mean Ao = 10,000° to a temperature of about 14,000°. But there is still a discrepancy. It may be remarked in this connection that if we assume mean Ao = 13,000° K then Fowler and Milne's formulæ give $P_e = 1{\cdot}1 \times 10^{-2}$ atmospheres and for B2 (maximum of He I) T = 20,300° K, a value quite consistent with the above observations. In fact, the discrepancy would now be the other way. The general body of evidence, however, seems to be in favour of the lower pressures.† It may also be noted that Fowler and Milne assume that the Balmer Series reaches maximum strength at type Ao. Menzel ‡ has recently placed the maximum at A5, and if this were correct it would affect Fowler and Milne's results. In fact, for the maximum at A3 the discrepancy would again go the other way. Menzel's result, however, is based on limited material and cannot be regarded as conclusive.

There is also the possibility of a change of pressure with type to be considered. A change in the value of P_e from $1{\cdot}3 \times 10^{-4}$ atmospheres at Ao to $5{\cdot}9 \times 10^{-6}$ at B2 would place B2 at 14,000° whilst keeping Ao at 10,000°. There is, in fact, some evidence in favour of a decreasing pressure as we proceed to the earlier types. King gives for the mean

* *M.N.*, **83**, 1923 May; **84**, 1924 May.

† The evidence is summarised by Miss Payne, *Stellar Atmospheres*, ch. III.

‡ *Harvard Circular*, 258, 1924.

colour indices of types Bo, B1, B2, the values $-0^{m}\cdot29$, $-0^{m}\cdot28$, $-0^{m}\cdot34$ respectively, the mean colour index being according to his results a minimum at B2. If these results be correct they indicate that the mean effective temperature of types Bo and B1 is actually less than in the case of type B2, and an application of the Fowler-Milne theory leads us to the conclusion that the pressure decreases rapidly as we pass from B2 to the earlier types.

Considerations such as these indicate that the matter cannot be regarded as finally settled. If the temperatures derived by Fowler and Milne be taken as substantially correct, the evidence points to a departure of the radiation received from the early type stars from the "black-body" distribution of radiation corresponding to the temperatures of their reversing layers.

It will be noticed that the temperature derived for ζ Persei* is abnormally low, that is, the star is abnormally red. This result is in agreement with its colour index, and does not seem to be very much in doubt. It will, however, be kept on the observing list. γ Cassiopeiæ also comes out low. It may be mentioned with regard to this star that for three comparisons the measures have been taken throughout the spectrum as well as at the ends. No trace has been found of any departure from the expected slope of the curve; in each case the plot of Δm_{λ} against $1/\lambda$ has been a straight line within the error of observation.

It has been stated above that the probable error of a single comparison is $\pm$ 0·09 in units of the gradient. This accidental error is made up of three parts, viz. (1) error of measurement, (2) error of photographic registration, (3) accidental variation in the transparency of the sky. In twenty cases the results of the two measurers were reduced separately, and the mean discordance, irrespective of sign in gradient, is ·035, indicating a probable error of measurement of $\pm$ ·015, for the mean result from the two observers. That is to say, the portion of the total probable error contributed by the measurement is relatively small. It should be noticed that since the two measurers measure different lateral spectra this probable error of $\pm$ ·015 includes part of the accidental error of photographic registration. We consider † that a considerable part of the total probable error of $\pm$ ·09 has its origin in variations of atmospheric transparency.

Summary.—The relative effective temperatures of twenty-two early-type stars have been determined. The results show a good correlation with colour index data. They conflict with the results deduced by Fowler and Milne from considerations of line intensity. Attention is drawn to ζ Persei, which comes out abnormally cool, that is, abnormally red.

* This star is classified as a normal B1 star in the Draper catalogue. Photographs of its spectra were obtained at Cambridge in 1921 by Mr. Stratton, who has drawn our attention to the variability of some of the lines. Mr. Baxandall has made a detailed examination of these photographs and informs us that according to the Harvard criteria the spectral type varies between B1 and B2. The star evidently merits further spectroscopic study.

† From considerations not mentioned in this paper.

Spectroscopic Parallaxes of B-type Stars (Fourth Paper): The Method of Adams and Joy. By D. L. Edwards, A.R.C.S., D.I.C.

(*Communicated by the Director of the Norman Lockyer Observatory.*)

I. *Introductory.*

It was pointed out in an earlier paper on this subject * that the method hitherto used at the Norman Lockyer Observatory for determining spectroscopic parallaxes of B-type stars was not applicable to a large number of these stars, particularly those of the later subdivisions. The most promising line of attack for general application to these early-type stars seems to be that devised by Adams and Joy,† based on the correlation between spectral type and absolute magnitude, together with a separation of the stars into two groups according to the sharpness or diffuseness of their spectral lines. In the case of the A-type stars this method has been criticised by Lindblad,‡ and later by Miss Fairfield,§ mainly on the grounds of a systematic difference between the Harvard classification and that of Adams and Joy. It is shown that stars in the Mount Wilson "*s*" group (sharp lines) are classified consistently slightly later in spectral type than at Harvard, while the "*n*" stars are allotted to a slightly earlier type. Owing to the rapid change in absolute magnitude with spectral type for the A stars, the magnitudes assigned to the *s* groups would therefore seem to be too faint, and the *n* groups would be made too bright. The result is that if the stars are classified according to the Harvard system and then divided into the Mount Wilson groups *n* and *s*, the *n* stars are found to be slightly brighter than the *s* stars of the same spectral subdivision—a result actually the reverse of the Mount Wilson supposition. In addition, Miss Fairfield has been unable to find sufficient evidence, from trigonometric parallaxes, of a correlation between absolute magnitude and width of the hydrogen lines as measured at Harvard. This does not necessarily vitiate the final results of Adams and Joy, since a correlation undoubtedly seems to exist between the groups of their classification and absolute magnitude; it has been suggested by Lindblad (*loc. cit.*) that the Mount Wilson classification of A-type stars may have the character of a luminosity classification, analagous to one based on the intensities of enhanced lines in the later types. The validity, however, of the use of line character as a criterion of absolute magnitude may be affected; and it seems advisable to analyse the results of Adams and Joy for B-type stars in order to ascertain if similar criticisms of the method may still be possible when it is used for these stars.

The following table is compiled from the Mount Wilson results for B-type stars. The stars are arranged according to the Harvard classi-

* *Monthly Notices, R.A.S.*, **85**, 440.
† *Ap. J.*, **58**, 242; *ibid.*, **57**, 294.
‡ *Ap. J.*, **59**, 305.
§ *Harvard Coll. Obs. Circular*, No. 264.

fication (column 1), each subdivision being split up into the two Mount Wilson groups, *s* and *n*. Under each of these two groups is given the mean Mount Wilson type, the number of stars in the group, and the mean absolute magnitude. The last two columns give the difference ($s - n$) for spectral type and for absolute magnitude between the two groups.

TABLE I.

Analysis of Mount Wilson Results.

Type. (H.D.)	*s*			*n*			*s–n*	
	Mean M.W. Type.	No. of Stars.	Mean M.	Mean M.W. Type.	No. of Stars.	Mean M.	Spec. Type.	M
B0	B0·4	5	−2·5	B0·6	5	−2·7	−0·2	+0·2
B1	B1·3	7	−2·1	B1·3	5	−1·8	0·0	−0·3
B2	B2·2	5	−1·9	B2·4	5	−1·5	−0·2	−0·4
B3	B3·0	21	−1·6	B3·5	46	−0·9	−0·5	−0·7
B5	B5·7	16	−1·0	B6·5	28	−0·3	−0·8	−0·7
B8	B8·5	15	−0·3	B8·0	36	+0·3	+0·5	−0·6
B9	B8·9	9	−0·3	B8·9	27	+0·4	0·0	−0·7

It will be noticed that although there is a slight systematic difference in classification between the two groups, yet this difference is of opposite sign to the difference found for A-type stars—that is, the *s* groups are classified earlier than the *n* groups. As might be expected, therefore, it is seen in the last column that (except for the sub-class B0) a pronounced difference of absolute magnitude of the required sign still remains in spite of the reclassification under Harvard types. The criticisms of Lindblad and Miss Fairfield thus appear not to hold for the B-type stars, and it would seem that there is some ground for believing in the reality of the relation between line character and absolute magnitude for these stars. This is further strengthened by a consideration of group motion parallaxes. If all the stars in the Mount Wilson list for which such parallaxes are available are arranged in the two groups of *n* and *s* stars, the mean absolute magnitudes are − 1·0 for the *s* group and − 0·6 for the *n* stars. The distribution in spectral type will have no relative effect on these magnitudes since this distribution is very much the same in each group ; the mean values of the Harvard types being B4·1 and B4·0 for the *s* and *n* groups respectively. The *s* stars are therefore, in the average, 0·4 magnitudes brighter than the *n* stars—a result in good agreement with the above table.

Finally, it may be pointed out that the quality of sharpness in a line is rather uncertain and may depend to some extent on the instrument employed (whether slit spectrograph or objective prism) and on the individual observer's conception of sharpness or his method of measurement (when mechanical means are employed). This has already been remarked by Schalen,* whose classification according to

* *Upsala Meddelanden*, No. 10.

sharpness of hydrogen lines shows a more pronounced correlation with reduced proper motion than Miss Fairfield's measurements of line widths; and although his classification has no striking relation with those of Adams and Joy or Abetti, yet his final results are in fairly good agreement with the latter observers.

Considerations such as these make it seem probable that a useful purpose might be served by applying this method of Adams and Joy (or a modification of it) to the objective prism spectra of B-type stars obtained at the Norman Lockyer Observatory. A preliminary examination and classification of these negatives yielded encouraging results and led to a more detailed, systematic investigation. The methods used, and the results, are described in the following sections of this paper. The spectra were all taken at Sidmouth with the 12-inch prismatic camera of the observatory, or with the same prism used by Dr. F. McClean at the Cape. The prism has an angle of 20°, and the dispersion at the focus of the 12-inch objective is such that the distance between Hβ and Hδ is 21·4 mm. The plates used at Sidmouth were chiefly Marion's "Record" plates or Ilford "Iso-Zenith," except in the case of some earlier photographs for which Barnet "Super-speed Ortho" were used. No appreciable differences were found between spectra of the same star taken on different plates. One setting of the focussing adjustments was used for all the photographs, as it was found that the definition was sufficiently good over the whole region of the spectrum chiefly used for this work (λ4481 to Hδ). The classifications, both of spectral type and of line characteristics, were made by eye estimations; comparisons being made with standard negatives.

II. *Classification of B-type Stars.*

(a) *Spectral Type.*

Since the method is based primarily on the correlation between spectral type and absolute magnitude, it is evident that the classification of the spectra should be as accurate as possible. The Harvard classification, while giving a valuable and mainly accurate system, is not in detail sufficiently reliable for this purpose, and requires to be checked by a reclassification of the stars under consideration. In such a comprehensive work as the Henry Draper Catalogue, it is not surprising (especially when the small scale of the spectra used is borne in mind) that the classification of individual stars should occasionally need revision. A change in the assigned spectral type of a star by even one subdivision would have an appreciable effect on the deduced absolute magnitude, and changes greater than this were occasionally found to be necessary. An extreme example may be mentioned in the case of ψ Eridani, a star of magnitude 4·8 classed as B8 in the Henry Draper Catalogue, but whose spectrum, judging from the Sidmouth photographs, can certainly not be placed later than B3. The total absence, also, of stars in the subdivisions B4, B6, and B7 suggests either that the Harvard sub-classes do not represent equal stages of progression from Bo to Ao (as was assumed by Adams and Joy), or that

stars strictly belonging to these missing groups have for some reason not been separated from the adjacent groups. Finally it may be remarked that there is a considerable range of variation in spectral type within one Harvard sub-class. For example, stars of type B3 show very appreciable differences among themselves, some approaching very closely to the B5 type, others to B2. Similar variations are to be found among other sub-classes. It is possible, in fact, by suitable selection, to find two stars of different Harvard types whose spectra are almost indistinguishable, while another pair with a common classification may show wide differences.

An examination was therefore made of all the B-type spectra which were available, with a view to their reclassification. It was decided to follow the Harvard system as closely as possible, by endeavouring to make the revised sub-types agree in the mean with the original Harvard values. For this reason the Mount Wilson classification could not be used, since in it the Harvard type B5 was transferred to B7 and the other groups adjusted accordingly. It was pointed out in some earlier work on the relation between line intensities and spectral type * that the shape of the resulting curves suggested that equal changes in the suffixes representing sub-types represented approximately equal stages of spectral sequence, thus affording further reason for adhering as closely as possible to the Harvard framework. It soon became evident that the sub-classes B4, B6, and B7 might usefully be interpolated to cover some of the variations in the adjacent groups, and a scheme was prepared describing spectra which could be taken as typical of the new groups. The most useful lines for this purpose in the Sidmouth spectra were the Helium lines at λλ4472, 4144, 4121 ; the Mg^+ line λ4481 ; and the Silicon double λλ4128, 4131. The relative intensities of these lines, as well as the general prominence or otherwise of the Hydrogen, Helium, and Calcium lines, were used in the classification which is described below. In this scheme the main characteristics of the new classes are described, together with remarks, taken from the Henry Draper Catalogue, on the adjacent types B3, B5, and B8 for the sake of comparison. Additional remarks are given in brackets after the Harvard descriptions.

Class B3.—Hydrogen lines about 0·5 as intense as in α CMa. Helium lines more prominent than in class B2 owing to extreme faintness of some O, Si, and C lines [4472 much stronger than 4481 ; 4121 stronger than Si double].

Class B4.—Helium lines less prominent than in class B3 ; 4472 stronger than 4481 ; Si double approximately equal to 4121 and fainter than 4144.

Class B5. —Increased intensity in [K] line and in Si double, which is stronger than 4121, but fainter than 4144. Line 4481 0·7 times as intense as 4472. [Calcium line [K] usually just visible.]

Class B6.—Si double much stronger than 4121, nearly equal to 4144 ; 4472 slightly stronger than 4481 ; Hydrogen lines increased slightly in intensity, Helium lines less prominent.

* *Monthly Notices, R.A.S.*, **83**, 50.

Class B7.—Si double equal to 4144; 4121 scarcely visible; 4481 equal to 4472; 4026 and 4472 easily seen, other Helium lines faint or invisible; [K] much fainter than 4026.

Class B8.—Helium lines 4026 and 4472 are present, together with several lines prominent in class A0; [K] is less intense than 4026; [4472 slightly fainter than 4481; Si double stronger than 4144].

For convenience the above remarks may be summarised in tabular form, showing the variations in intensity of the chief lines concerned, viz. the pair 4481, 4472; the Si double and adjacent lines; and the [K] line of Calcium. In Table II. the symbols $>>$, $>$, and $\gtrless$ are used to indicate "much stronger," "stronger," and "slightly stronger," with corresponding meanings for the reversed symbols.

TABLE II.

Principal Lines used for Classification.

Type.	4472 and 4481.	Si Double and He Group.	Calcium [K].	Remarks.
B3	He $>>$ Mg	4121 $>$ Si $<<$ 4144	Usually invisible	H and Ca increasing; He decreasing in prominence from B3 to B8.
B4	He $>$ Mg	4121 $=$ Si $<$ 4144	Faint or invisible	
B5	He $>$ Mg	4121 $<$ Si $<$ 4144	Faint	
B6	He $>$ Mg	4121 $<<$ Si $<$ 4144	$<<$4026	
B7	He $=$ Mg	.. Si $=$ 4144	$<<$4026	
B8	He $<$ Mg	.. Si $>$ 4144	$<$4026	

In spite of occasional slight differences in the classification as indicated by different lines, there was usually little difficulty in deciding to which class a star should be assigned; and when the classification was completed it was found to be in good agreement, in the mean, with the Harvard values, as shown in the following summary:—

Revised Class:	B0	B1	B2	B3	B4	B5	B6	B7	B8	B9
Mean H.D. Type:	B0	B0·8	B1·9	B3·0	B3·5	B4·7	B6·1	B7·8	B8·2	B8·9

(b) *Line Character.*

The above classification was performed without regard to the sharpness or diffuseness of the lines, except in so far as this quality affected the apparent intensities of lines used in classifying. The negatives were therefore re-examined from the point of view of line characters. It was at once evident that great differences in the sharpness of the spectral lines exist among the B-type stars; but these differences appeared to show a continuous gradation from very sharp to very nebulous types, and it was difficult to divide the stars into two groups, one with sharp and the other with nebulous lines, as was done by Adams and Joy. It was equally difficult to devise a numerical method of describing this quality except by actual measurements of line widths, which did not give encouraging results. A compromise was therefore decided upon, in which the spectra were arranged in five

groups according to the sharpness of their lines, these groups being designated by the letters

ss, *s*, *ns*, *n*, *nn*

in order of decreasing sharpness. Any further refinement was found to be impracticable without some mechanical method of measuring the sharpness of spectral lines, since the transition from one of the above groups to an adjacent group represented a difference in sharpness only just appreciable when estimated by eye. The extreme groups, *ss* and *nn*, seem to be exceptional, and are not represented in some of the spectral classes. A large proportion of the stars examined belong, therefore, to the three intermediate groups. The *ns* group represents the average character of the lines in a spectral class; stars containing lines slightly sharper or more diffuse being classed as *s* or *n*, and (when the difference is very pronounced) as *ss* or *nn*. It was found that the average sharpness of the lines appeared to decrease in proceeding from B0 to B9 stars, so that it was necessary to have different standards of sharpness for the different classes. This was also necessitated to some extent by the varying intensities of the lines in different spectral types, making it difficult, for example, to compare directly a B5 star with one of type B1. A set of five standard negatives was therefore chosen, when possible, for each spectral class, to represent the different line characters in that class. All available negatives were then compared with these standards and arranged according to their line characters in the five groups described above.

The results of this classification, both for spectral type and sharpness of the lines, are given in Table V., columns 7 and 8.

III. *The Reduction to Absolute Magnitude.*

(a) *General Considerations.*

In order to express the relation between absolute magnitude and the stellar classification outlined above, the five line-character groups were considered separately, and curves deduced for each of them showing the relation between spectral type and absolute magnitude. The chief difficulty lay in the small number of stars present in some of the groups. The 300 stars examined had to be divided into about 38 groups with an average of about 8 stars to a group; the actual number in some cases being considerably less than this. An attempt was therefore made to utilise (in addition to the material discussed later) parallaxes and absolute magnitude data relating to stars not included in this investigation, or to the B-type stars generally.

The correlation between spectral type and absolute magnitude has been determined in several ways by different investigators, notably by Charlier,* Malmquist,† Plummer,‡ Kapteyn,§ and others. In addition, trigonometrical parallaxes, parallactic motions, and cluster parallaxes from the Perseus, Pleiades, Scorpio-Centaurus, and Orion

* *Lund Meddel.*, Series II., No. 14.
† *Ibid.*, No. 22.
‡ *Monthly Notices, R.A.S.*, **73**, 185.
§ *Ap. J.*, **47**, 255.

clusters, afford valuable data for determining this relation with some accuracy for the B-type stars in general. Mean absolute magnitudes from the above sources were plotted against spectral class (irrespective of line characteristics) and a smooth curve drawn expressive of the required relation. It was assumed that this curve represented the normal condition for B-type stars and approximated very closely to the curve required for the "*ns*" group. This assumption implies that the Harvard spectral types agree in the mean with the revised types used in this paper (which appears to be justified by the comparison given above), and also that there is not a preponderance of either sharp or nebulous line stars among those which were used in determining this curve. It was necessary to restrict this investigation to stars within the same range of apparent magnitude as those in the present list. Owing to the dispersion in absolute magnitude the inclusion of fainter apparent magnitudes would introduce a larger proportion of faint absolute magnitudes, and the results would not be applicable to the stars under consideration. The results obtained in this way were not used directly, but were combined with the data described in the following paragraphs and used in obtaining the final reduction curve for stars of the "*ns*" group.

(b) *Parallactic Motions.*

The method of most general application to the actual stars investigated in the present paper is that of parallactic motion. Most of the stars are given in Boss's Catalogue, and the proper motions were taken from this source, corrected according to the formulæ given in the introduction. The position of the Solar Apex was taken as

$$\text{R.A. } 18^{h}\ 0^{m} \qquad \text{Dec. } +34^{\circ},$$

and some of the initial computations were much simplified by the use of Smart's charts, based on the same position of the apex. The solar velocity was taken as 21 km./sec. Stars within 30° of the apex were omitted from these calculations in order to avoid undue influence from errors in the assumed position of the apex, and in addition a few stars with abnormal proper motions or other pronounced peculiarities were discarded. The remaining stars were grouped according to their classifications and the mean parallax of each group found by the usual formulæ. Radial velocities and peculiar motions were not used for obtaining mean parallaxes, since it has been shown by Russell * that for B-type stars parallactic motions should give better results than peculiar motions, the latter being possibly vitiated by the presence of undetected spectroscopic binaries, causing the resulting parallaxes to be too small. From the mean parallax, $\bar{\pi}$, the mean absolute magnitude, $\bar{M}$, of each group was deduced by the formula previously derived for a similar purpose, viz.—

$$\bar{M} = 5 \log \{n/\Sigma 10^{-0.2m}\} + 5 + 5 \log \bar{\pi},$$

n being the number of stars in the group, and m the individual apparent magnitudes.

* *Ap. J.*, **54**, 140.

Many of the groups contained an insufficient number of stars to be treated in this way, but the number of groups for which mean absolute magnitudes could be obtained was greater than that for any of the other individual methods described later. For this reason it is of interest to tabulate the results obtained from parallactic motions alone, since the homogeneity of the results affords a useful test of the spectroscopic criteria. These results, as shown in Table III., appear to demonstrate the relation between absolute magnitude and line sharpness. The decrease of absolute magnitude in passing from *ss* stars to *nn* stars is well marked in every spectral type, for which results are available. This may be due partially to chance, since in some of the smaller groups the mean absolute magnitude is somewhat uncertain, and may be largely dominated by one star. The possibility of systematic error, also, on account of group motion or an erroneous value of the solar velocity, or from some other cause, cannot be excluded. Even after due allowance has been made, however, for such contingencies, there seems to be strong evidence in favour of the correlation, even though the actual figures given below may need systematic correction.

Table III.

Mean Absolute Magnitudes from Parallactic Motions.

	ss.	*s.*	*ns.*	*n.*	*nn.*
B1	−3·7	−3·5	−2·6		
B2	−2·9	−2·7	−2·3	−1·9	
B3	−2·8	−2·3	−1·8	−1·5	−1·2
B4	..	−1·5	−1·2	−0·6	
B5	−2·0	−1·6	−1·1	−0·8	
B6	..	−1·7	−0·8		
B7	..	−1·5	−0·7		
B8	..	−1·0	−0·7	−0·2	
B9	..	−0·6	−0·1	+0·1	

(c) *Moving Clusters.*

The most reliable individual parallaxes available for B-type stars are probably those derived from moving clusters. The values deduced by Rasmuson * were used, and the four principal clusters (Orion, Scorpio-Centaurus, Perseus, and Pleiades) were first considered separately, in order to test the possibility of systematic differences between the various groups. It was found that the Perseus cluster gave absolute magnitudes which were fainter than those deduced from any other methods, whereas for the Orion group this result was reversed. In deducing mean absolute magnitudes for different spectral types, parallaxes from all four of the above groups were combined and supplemented by some of Kapteyn's values for additional stars showing characteristics of group motion. In this way data of some sort were

* *Lund Meddel.*, Series II., No. 26.

available with respect to the absolute magnitudes of about 25 different spectral classes, though the average number of stars in each group was very small for the purpose of deriving mean magnitudes. Thus, out of the above 25 groups, only 11 were represented by more than 3 stars, and in a number of cases only a single star was found to represent a group. Owing, however, to the probable accuracy of cluster parallaxes, it was decided to use the results without any attempt to apply a system of arbitrary weights when combining them with results from other sources.

The final results from cluster parallaxes furnish additional evidence in favour of the change of absolute magnitude with line sharpness, though on account of the smaller number of results which were available this evidence is not quite so striking as that afforded by parallactic motions. If the stars are arranged in the five character groups (*ss* to *nn*) irrespective of spectral type, the following mean absolute magnitudes result from the moving cluster data :—

$$ss : -2{\cdot}6, \quad s : -1{\cdot}8, \quad ns : -1{\cdot}4, \quad n : -0{\cdot}7, \quad nn : -0{\cdot}6.$$

It must, however, be pointed out that the mean spectral types of these groups show a gradual change throughout the above series, which would at least partially account for the decrease in luminosity, though it could scarcely be sufficient to cover the entire change of two magnitudes in proceeding from *ss* to *nn*. In order to eliminate the effect of type, mean differences were formed between certain character groups for stars of the same spectral type. It was found that the mean value of *ss–ns* (in absolute magnitude) was $-0{\cdot}9$ for stars of types B0 to B3 ; and the mean *s–n* was $-0{\cdot}7$ for types B2 to B4. A decided drop in luminosity is thus associated with loss of sharpness of the spectral lines. Again, the evidence from stars of type B3 is very strong. More cluster parallaxes were available for stars of this type than for any other, and it is the only type in which mean absolute magnitudes were found for all of the five character groups. The following are the values of these magnitudes for type B3 stars :—

$$ss : -2{\cdot}3, \quad s : -1{\cdot}7, \quad ns : -1{\cdot}4, \quad n : -0{\cdot}8, \quad nn : -0{\cdot}7.$$

The correlation between line character and luminosity is at once evident.

Finally, it may be suggested that the following is a legitimate method of treating these cluster magnitudes. If the correlation between spectral type and absolute magnitude be assumed (as may reasonably be done) then absolute magnitudes from cluster parallaxes may be used to show this relation for each of the line-character groups independently. Curves may be drawn through the points thus obtained and corrected values of the original absolute magnitudes read off. In this way irregularities due to errors in the parallaxes or to scarcity of material may be smoothed out, and additional values of absolute magnitudes may be interpolated. When this was done with the available data, the corrected and interpolated mean absolute magnitudes showed a strong resemblance to the results from parallactic motions given in Table III. The decrease in brightness with decrease of line

sharpness was well marked, without exception, for each spectral type, though the actual values were on the average about 0·5 magnitude fainter than those in Table III. (*N.B.*—The original values of the cluster magnitudes, and not the above corrected values, were used in preparing the final curves in combination with other data as explained later.)

(d) *Parallaxes from Trigonometric and other Sources.*

Trigonometric parallaxes are not sufficiently trustworthy or numerous to be of very great value in forming mean parallaxes for the small groups under consideration. The only method of utilising these parallaxes is to consider them in relation to each of the five character groups independently of spectral type. If the mean is taken of the trigonometric parallaxes available in each group, together with the mean spectral type of the same stars, then one point will be obtained on each of the five standard curves described below. Even when used in this way, however, the number of stars in some of the groups is small, and the addition or omission of one star (especially with a large negative parallax) has considerable effect on the mean of the group. During the course of this investigation, a paper was published by the Astronomer Royal * on a method of correcting trigonometric parallaxes from the frequency curve of the values given in Schlesinger's Catalogue, by means of which negative parallaxes are given the most probable positive value and large positive parallaxes are reduced. These corrected parallaxes of Sir Frank Dyson gave mean absolute magnitudes rather fainter than those derived directly from the original values, but they showed more regularity, and indicated a decided decrease in luminosity with decreasing sharpness of the lines. The following values were finally adopted from trigonometric sources :

$$B2{\cdot}1ss : -2{\cdot}5, \quad B2{\cdot}3s : -2{\cdot}1, \quad B3{\cdot}4ns : -1{\cdot}6,$$
$$B3{\cdot}5n : -1{\cdot}0, \quad B3{\cdot}5nn : -0{\cdot}3.$$

In addition to the above sources, a few points on the standard curves were deduced from the statistical work of the authorities previously mentioned, whenever it was applicable to the actual stars in the present list.

(e) *The Standard Curves and Final Results.*

The results enumerated in the preceding paragraphs were all collected and used in forming standard curves showing the relation between absolute magnitude and spectral type. The five line-character groups were considered separately, and smooth curves drawn through the points available in each case. Finally, these five curves were assembled in one diagram (see fig. 1), which showed by the regular displacement of the curves the relation between line sharpness and absolute magnitude. No final adjustment of the curves was found to be necessary after their assembly, and, as will be seen from the diagram, the regular progression in luminosity from the *ss* to the *nn* curves is very pro-

* *Monthly Notices, R.A.S.*, **86**, 686.

nounced. From the curves thus obtained, mean values of absolute magnitude were read off for the various groups required, and these

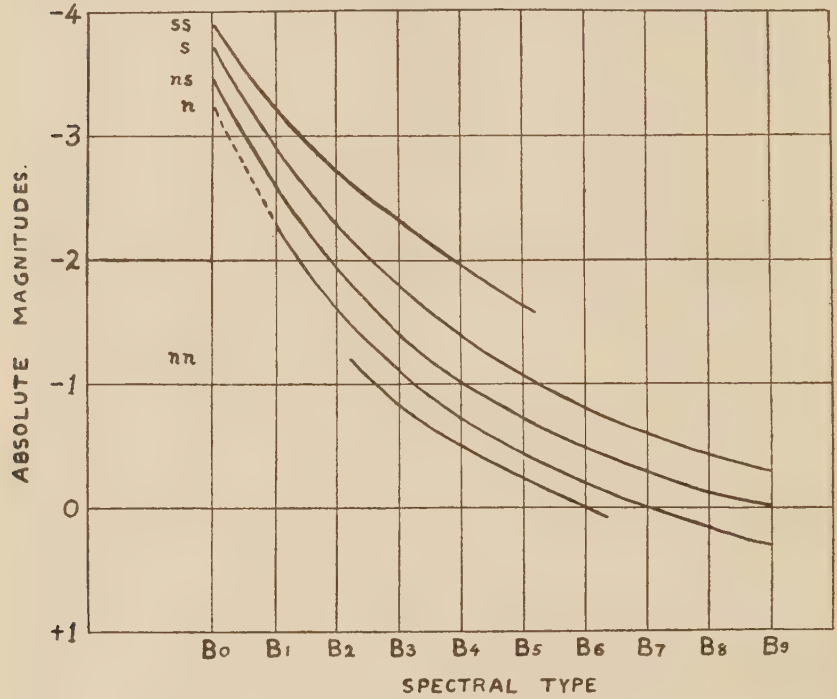

FIG. 1.—Assembly of curves showing relation between Luminosity and Type for different line characters.

results (given in Table IV.) were finally used in deducing parallaxes for individual stars from their classification.

TABLE IV.

Mean Absolute Magnitudes of Different Groups.

	ss.	*s.*	*ns.*	*n.*	*nn.*
B0	−3·9	−3·7	−3·4	(−3·2)	
B1	−3·2	−2·9	−2·6	−2·3	
B2	−2·7	−2·3	−1·9	−1·6	
B3	−2·3	−1·8	−1·4	−1·1	−0·8
B4	−2·0	−1·3	−1·0	−0·7	−0·5
B5	−1·6	−1·0	−0·7	−0·5	−0·3
B6	..	−0·8	−0·5	−0·3	−0·1
B7	..	−0·6	−0·2	0·0	
B8	..	−0·5	−0·1	+0·2	
B9	..	−0·3	0·0	+0·3	

The final results for the absolute magnitudes and parallaxes of 300 B-type stars are given in Table V. The columns in this table have the following significance :

1. Henry Draper Catalogue number.
2. Name or constellation.
3, 4. Position for 1900.
5. Apparent magnitude.
6. Spectral type (H.D.).
7, 8. Revised spectral type and line-character group.
9, 10. Absolute magnitude and parallax from classification.
11, 12. Other parallaxes for comparison, with authorities.

The abbreviations used in column 12 are as follows : W = Mount Wilson, t = trigonometric, g = group motion, d = dynamical. The units in column 11 are thousandths of a second of arc.

TABLE V.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
		1900.			Spectrum.					Other Parallaxes.	
H.D.	Name.	R.A.	Dec.	*m.*	H.D.	N.L.O.		M.	π.	π.	Auth.
		h m	° ′						″		
144	10 Cas.	0 1·2	+63 37	5·5	B8	B8	*ns*	−0·1	0·008	10	W.
560	34 Psc.	4·9	+10 36	5·5	B8	B9	*ns*	0·0	·008	10	W.
886	γ Peg.	8·1	+14 38	2·9	B2	B3	*s*	−1·8	·012		
1279	−And.	11·9	+47 24	5·8	B9	B7	*s*	−0·7	·005		
1337	−Cas.	12·5	+50 53	6·1	B0	B1	*n*	−2·3	·002		
1976	−Cas.	18·9	+51 28	5·4	B3	B3	*ns*	−1·4	·004	6	W
2729	13 Cas.	25·7	+65 58	6·1	B5	B6	*ns*	−0·5	·005		
2772	λ Cas.	26·2	+53 59	4·9	B8	B9	*n*	+0·3	·012	14	g.
2905	$\varkappa$ Cas.	27·3	+62 23	4·2	B0	B0	*ss*	−3·9	·002		
3240	−Cas.	30·5	+53 38	5·1	B5	B7	*ns*	−0·2	·009	10	W.
3360	ζ Cas.	31·4	+53 21	3·7	B3	B3	*ss*	−2·3	·006	−21	t.
3369	π And.	31·5	+33 10	4·4	B3	B5	*n*	−0·5	·010	− 8	t.
3379	53 Psc.	31·6	+14 41	5·9	B3	B3	*s*	−1·8	·003	3	W.
3901	ξ Cas.	36·5	+49 58	4·9	B3	B4	*ns*	−1·0	·007		
4142	−Cas.	38·8	+47 19	5·6	B3	B3	*n*	−1·1	·005		
4180	*o* Cas.	39·2	+47 44	4·7	B2	B3	*ns*	−1·4	·006		
4382	23 Cas.	41·1	+74 18	5·4	B8	B7	*s*	−0·6	·006	8	W.
4636	ν Cas.	43·2	+50 25	5·0	B9	B7	*ns*	−0·2	·009	9	g.
4727	ν And.	44·3	+40 32	4·4	B3	B4	*ns*	−1·0	·008		
6300	−Cas.	58·9	+50 29	6·5	B3	B4	*ns*	−1·0	·003		
6811	φ And.	1 3·7	+46 42	4·3	B8	B8	*ns*	−0·1	·013	11, 9	t, d.
6972	32 Cas.	5·1	+64 29	5·5	B8	B9	*ns*	0·0	·008	9	W.
7374	87 Psc.	8·8	+15 36	5·9	B8	B8	*ns*	−0·1	·006	6	W.
10205	τ And.	34·7	+40 4	4·9	B8	B8	*s*	−0·5	0·008		

TABLE V.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
H.D.	Name.	1900. R.A.	1900. Dec.	*m.*	Spectrum. H.D.	Spectrum. N.L.O.		M.	π.	Other Parallaxes. π.	Other Parallaxes. Auth.
		h m	° ′						″		
10390	—Tri.	36·3	+34 44	5·5	B8	B8	*ns*	−0·1	0·008	8, 13	W, g.
11241	1 Per.	45·4	+54 39	5·5	B3	B3	*n*	−1·1	·005	5, 9	W, g.
11415	ε Cas.	47·2	+63 11	3·4	B3	B4	*ns*	−1·0	·013	17, 11, 1	W, g, t.
11529	ω Cas.	48·2	+68 12	5·0	B8	B6	*ns*	−0·5	·008	8	W.
12303	4 Per.	55·6	+54 1	5·0	B8	B7	*ns*	−0·2	·009	11	W.
14228	φ Eri.	2 12·9	−51 59	3·8	B8	B6	*n*	−0·3	·015		
14372	—And.	14·2	+46 51	6·1	B5	B5	*ss*	−1·6	·003	4	W
16582	δ Ceti.	34·4	− 0 6	4·0	B2	B3	*ss*	−2·3	·005		
16727	11 Per.	35·8	+54 41	5·7	B8	B6	*ns*	−0·5	·006		
17036	*o* Ari.	39·0	+14 53	5·8	B8	B8	*ns*	−0·1	·007		
17081	π Cet.	39·4	−14 17	4·4	B5	B6	*ns*	−0·5	·011		
17543	π Ari.	43·7	+17 3	5·3	B5	B5	*ss*	−1·6	·004	7	W.
17573	41 Ari.	44·1	+26 51	3·7	B8	B8	*ns*	−0·1	·017		
17769	σ Ari.	46·0	+14 40	5·5	B5	B5	*n*	−0·5	·006	7	W.
18604	λ Cet.	54·4	+ 8 31	4·7	B5	B6	*s*	−0·8	·008		
20336	—Cam.	3 11·2	+65 17	4·8	B3*p*	B3	*nn*	−0·8	·008		
20365	29 Per.	11·5	+49 51	5·3	B3	B4	*ns*	−1·0	·006	6, 11	W, g.
20418	31 Per.	12·0	+49 43	5·1	B3	B3	*n*	−1·1	·006	7, 10	W, g.
20756	τ Ari.	15·5	+20 47	5·2	B3	B5	*ns*	−0·7	·007		
20809	—Per.	16·1	+48 51	5·3	B3	B4	*n*	−0·7	·006	7, 9	W, g.
21278	—Per.	20·9	+48 43	4·9	B5	B5	*ns*	−0·7	·008	7, 8	W, g.
21362	—Per.	21·7	+49 31	5·6	B5	B5	*n*	−0·5	·006	7, 12	W, g.
21364	ξ Tau.	21·7	+ 9 23	3·8	B8	B7	*n*	0·0	·017		
21428	34 Per.	22·2	+49 10	4·7	B5	B5	*ns*	−0·7	·008	8, 11, 13,	W, g, t.
21933	6 Tau.	27·1	+ 9 2	5·6	B8	B9	*n*	+0·3	·009		
22192	ψ Per.	29·4	+47 51	4·3	B5*p*	B5	*nn*	−0·3	·012	11	g.
22203	19 Eri.	29·4	−21 58	4·3	B8	B8	*ns*	−0·1	·013		
22928	δ Per.	35·8	+47 28	3·1	B5	B5	*n*	−0·5	·019	20, 11, 5	W, g, t.
22951	40 Per.	36·0	+33 39	5·0	B2	B2	*s*	−2·3	·003	3	W.
23180	*o* Per.	38·0	+31 58	3·9	B1	B2	*ns*	−1·9	·007	13,	t.
23302	17 Tau.	39·0	+23 48	3·8	B5*p*	B6	*ns*	−0·5	·014	19, 14, 20	W, g, t.
23338	19 Tau.	39·3	+24 10	4·4	B5	B6	*ns*	−0·5	·011	13, 13	W, g.
23363	24 Eri.	39·5	− 1 29	5·1	B8	B7	*ns*	−0·2	·009	12	W.
23408	20 Tau.	39·9	+24 4	4·0	B5	B6	*s*	−0·8	·011	14, 14	W, g.
23480	23 Tau.	40·4	+23 39	4·3	B5*p*	B5	*ns*	−0·7	·010	12, 15	W, g.
23630	η Tau.	41·5	+23 48	3·0	B5*p*	B7	*ns*	−0·2	·023	29, 13, 7	W, g, t.
23850	27 Tau.	43·2	+23 45	3·8	B8	B6	*ns*	−0·5	0·014	22, 14, −26	W, g, t.

TABLE V.—*continued.*

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8. Spectrum. N.L.O.	9. M.	10. *π.* ″	11. Other Parallaxes. *π.*	12. Other Parallaxes. Auth.
24398	ζ Per.	47·8	+31 35	2·9	B1	B1	*ss*	−3·2	0·006	−3	t.
24760	ε Per.	51·1	+39 43	3·0	B1	B1	*s*	−2·9	·007	10, 9	W, g.
25940	48 Per.	4 1·4	+47 27	4·0	B3*p*	B4	*nn*	−0·5	·013	11, 15	g, t.
26793	−Tau.	9·2	+ 9 46	5·2	B8	B9	*ns*	0·0	·009		
26912	μ Tau.	10·1	+ 8 39	4·3	B3	B4	*s*	−1·3	·008		
27192	−Per.	12·6	+50 41	5·5	B3	B3	*ns*	−1·4	·004	6	W.
27396	53 Per.	14·3	+46 16	4·9	B3	B4	*s*	−1·3	·006	11	g.
28149	72 Tau.	21·3	+22 46	5·4	*B*5	B5	*ns*	−0·7	·006	7	W.
29248	ν Eri.	31·3	− 3 33	4·1	B2	B2	*s*	−2·3	·005		
29763	τ Tau.	36·2	+22 46	4·3	B5	B5	*ns*	−0·7	·010	10, 8	W, t.
30211	μ Eri.	40·5	− 3 26	4·2	B5	B4	*ns*	−1·0	·009		
30614	α Cam.	44·1	+66 10	4·4	Bo	Bo	*ns*	−3·4	·003	4, −25	W, t.
30836	3 Ori.	45·9	+ 5 26	3·8	B3	B3	*ss*	−2·3	·006	6, 5, 0	W, g, t.
31237	8 Ori.	49·0	+ 2 17	3·9	B3	B3	*ss*	−2·3	·006	8, 6, 3	W, g, t.
32249	ψ Eri.	56·6	− 7 20	4·8	B8	B3	*ns*	−1·4	·006		
32964	66 Eri.	5 1·8	− 4 47	5·2	B9	B9	*n*	+0·3	·011		
33328	λ Eri.	4·4	− 8 53	4·3	B2	B2	*n*	−1·6	·007		
33802	ι Lep.	7·7	−11 59	4·5	B8	B6	*ns*	−0·5	·010		
34816	λ Lep.	15·0	−13 17	4·3	B1	B1	*s*	−2·9	·004	6	g.
35039	22 Ori.	16·7	− 0 29	4·7	B3	B3	*ns*	−1·4	·006	5, 5	W, g.
35149	23 Ori.	17·6	+ 3 27	5·0	B3	B3	*n*	−1·1	·006		
35411	η Ori.	19·4	− 2 29	3·4	B1	B2	*n*	−1·6	·010	7, 6	W, g.
35439	25 Ori.	19·6	+ 1 45	4·7	B3*p*	B3	*nn*	−0·8	·008		
35468	γ Ori.	19·8	+ 6 16	1·7	B2	B2	*ss*	−2·7	·013	18, 9, 19	W, g, t.
35532	113 Tau.	20·3	+16 36	6·2	B3	B3	*n*	−1·1	·004	4	W.
35671	115 Tau.	21·3	+17 53	5·3	B3	B4	*ns*	−1·0	·006		
35708	114 Tau.	21·6	+21 51	4·8	B3	B3	*ss*	−2·3	·004		
35715	ψ Ori.	21·6	+ 3 0	4·7	B2	B2	*ns*	−1·9	·005	8, 6, −12	W, g, t.
36267	32 Ori.	25·4	+ 5 52	4·3	B3	B5	*ns*	−0·7	·010	19	d.
36371	χ Aur.	26·2	+32 7	4·9	B1	B3	*ss*	−2·3	·004		
36486	δ Ori.	26·9	− 0 22	2·5	Bo	Bo	*s*	−3·7	·006	10, 5, 9	W, g, t.
36512	υ Ori.	27·1	− 7 23	4·6	B3	B1	*ns*	−2·6	·004	4, 6	W, g.
36576	120 Tau.	27·6	+18 29	5·5	B3*p*	B3	*nn*	−0·8	·006		
36819	121 Tau.	29·3	+23 58	5·3	B3	B3	*ns*	−1·4	·005		
36822	37 Ori.	29·3	+ 9 25	4·5	Bo	Bo	*s*	−3·7	·002	2	t.
37018	42 Ori.	30·4	− 4 54	4·7	B3	B3	*s*	−1·8	·005	6	W.
37128	ε Ori.	31·1	− 1 16	1·8	Bo	Bo	*s*	−3·7	0·008	14, 5, 5, 10	W, g, t, d.

TABLE V.—*continued.*

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8.	9. M.	10. *π*. ″	11. Other Parallaxes. *π*.	12. Other Parallaxes. Auth.
37438	125 Tau.	33·5	+25 50	5·0	B3	B4	*s*	−1·3	0·005		
37468	*σ* Ori.	33·7	− 2 39	3·8	Bo	B1	*ns*	−2·6	·005	5, 5, −9	W, g, t.
37711	126 Tau.	35·5	+16 29	4·9	B3	B3	*ns*	−1·4	·006	7	t.
37742	*ζ* Ori.	35·7	− 2 0	2·1	Bo	Bo	*ns*	−3·4	·008	11, 6, −19, 16	W, g, t, d.
37756	−Ori.	35·8	− 1 11	5·0	B3	B3	*n*	−1·1	·006	7, 6	W, g.
37795	*α* Col.	36·0	−34 8	2·8	B5*p*	B6	*ns*	−0·5	·022		
38622	133 Tau.	42·0	+13 52	5·2	B5	B4	*s*	−1·3	·005		
38771	*κ* Ori.	43·0	− 9 42	2·2	Bo	Bo	*s*	−3·7	·007	12, 6, 29	W, g, t.
38899	134 Tau.	43·9	+12 37	4·9	B9	B9	*s*	−0·3	·009		
39291	55 Ori.	46·5	− 7 33	5·3	B3	B3	*ns*	−1·4	·005		
40111	139 Tau.	51·8	+25 57	4·9	B2	B1	*s*	−2·9	·003		
40967	3 Mon.	57·1	−10 36	5·0	B8	B3	*s*	−1·8	·004		
41753	*ν* Ori.	6 1·9	+14 47	4·4	B2	B3	*s*	−1·8	·006		
42560	*ξ* Ori.	6·3	+14 14	4·4	B3	B3	*ns*	−1·4	·007		
42690	−Mon.	7·0	− 6 31	5·1	B3	B3	*s*	−1·8	·004		
43247	73 Ori.	10·1	+12 35	5·4	B9	B8	*s*	−0·5	·007	8	W.
44112	7 Mon.	14·9	− 7 47	5·1	B3	B3	*n*	−1·1	·006		
44743	*β* CMa.	18·3	−17 54	2·0	B1	B1	*s*	−2·9	·010	14, 12	W, t.
45542	*ν* Gem.	23·0	+20 17	4·1	B5	B6	*n*	−0·3	·013		
45546	10 Mon.	23·0	− 4 42	5·0	B3	B3	*ns*	−1·4	·005		
46487	−Mon.	28·6	− 1 9	5·0	B3	B4	*n*	−0·7	·007		
47670	*ν* Pup.	34·7	−43 6	3·2	B8	B8	*n*	+0·2	·025		
48879	42 Cam.	40·5	+67 41	5·0	B3	B3	*ns*	−1·4	·005		
48977	16 Mon.	41·1	+ 8 42	5·8	B3	B3	*ns*	−1·4	·004		
49340	43 Cam.	42·9	+69 0	5·1	B5	B5	*ns*	−0·7	·007		
50707	15 CMa.	49·2	−20 6	4·7	B1	B2	*s*	−2·3	·004		
52089	*ε* CMa.	54·7	−28 50	1·6	B1	B1	*ss*	−3·2	·011		
53244	*γ* CMa.	59·2	−15 29	4·1	B5	B8	*s*	−0·5	·012		
53974	−CMa.	7 2·0	−11 8	5·3	B3	B3	*ss*	−2·3	·003		
57821	−CMa.	17·8	−18 50	4·9	B8	B9	*s*	−0·3	·009		
58715	*β* CMi.	21·7	+ 8 29	3·1	B8	B8	*n*	+0·2	·026	20	t.
63975	*ζ* CMi.	46·5	+ 2 1	5·1	B8	B7	*s*	−0·6	·007		
67797	−Pup.	8 4·5	−18 57	4·3	B3	B5	*s*	−1·0	·009		
67880	−Pup.	4·9	−15 57	5·5	B3	B3	*s*	−1·8	·004	4	W.
74280	*η* Hya.	38·0	+ 3 46	4·3	B3	B3	*s*	−1·8	·006		
75333	14 Hya.	44·3	− 3 4	5·2	B9	B9	*ns*	0·0	·009		
78316	*κ* Cnc.	9 2·4	+11 4	5·1	B8	B8	*s*	−0·5	0·008		

TABLE V.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
H.D.	Name.	1900. R.A.	1900. Dec.	*m.*	Spectrum. H.D.	Spectrum. N.L.O.		M.	π.	Other Parallaxes. π.	Other Parallaxes. Auth.
		h m	° ′						″		
78556	19 Hya.	3·8	− 8 11	5·5	B8	B7	*ns*	−0·2	0·007	10	W.
79158	36 Lyn.	7·3	+43 38	5·3	B8	B8	*s*	−0·5	·007	11	W.
79931	24 Hya.	11·8	− 8 19	5·5	B9	B7	*ns*	−0·2	·007	11	W.
81188	$\varkappa$ Vel.	19·0	−54 35	2·6	B3	B3	*ns*	−1·4	·016	11	g.
83754	$\varkappa$ Hya.	35·5	−13 53	5·0	B3	B5	*n*	−0·5	·008	8	W.
87015	−Leo	57·2	+22 26	5·6	B3	B3	*n*	−1·1	·005		
87504	v^2 Hya.	10 0·3	−12 35	4·7	B8	B8	*n*	+0·2	·013		
89688	23 Sex.	15·9	+ 2 47	6·5	B3	B3	*ns*	−1·4	·003	3	W.
90882	δ Sex.	24·4	− 2 14	5·2	B9	B9	*ns*	0·0	·009	11	W.
90994	30 Sex.	25·2	− 0 8	5·0	B5	B5	*ns*	−0·7	·007	6	W.
91120	−Hya.	26·1	−13 5	5·5	B9	B9	*n*	+0·3	·009	8	W.
91316	ϱ Leo	27·5	+ 9 49	3·9	B*op*	B1	*s*	−2·9	·004	6, 29	W, t.
92938	−Car.	38·7	−63 57	5·2	B3	B5	*ns*	−0·7	·007		
93030	θ Car.	39·4	−62 52	3·0	B0	B1	*ns*	−2·6	·008		
93152	42 LMi.	40·3	+31 13	5·4	B9	B9	*ns*	0·0	·008	10	W.
93194	−Car.	40·5	−63 26	5·1	B3	B3	*ns*	−1·4	·005		
93607	−Car.	43·2	−63 52	5·1	B5	B3	*ns*	−1·4	·005		
100600	90 Leo	11 29·5	+17 21	5·8	B3	B4	*n*	−0·7	·005	5	W.
100841	λ Cen.	31·2	−62 28	3·3	B9	B9	*ns*	0·0	·022	17	g.
100889	θ Cra.	31·6	− 9 15	4·8	B9	B8	*n*	+0·2	·012	14	W.
105382	−Cen.	12 2·9	−50 6	4·8	B5	B4	*ns*	−1·0	·007	9	g.
105435	δ Cen.	3·2	−50 10	2·9	B3*p*	B3	*nn*	−0·8	·018	14	g.
109387	$\varkappa$ Dra.	29·2	+70 20	3·9	B5*p*	B5	*n*	−0·5	·013	− 3	t.
109668	α Mus.	31·2	−68 35	2·9	B3	B3	*ns*	−1·4	·014	15	g.
110879	β Mus.	40·1	−67 34	3·3	B3	B3	*ns*	−1·4	·012	14	g.
111123	β Cru.	41·9	−59 9	1·5	B1	B1	*ss*	−3·2	·012	18, 8	g, t.
113797	14 CVn.	13 1·1	+36 20	5·1	B9	B8	*ns*	−0·1	·009	13	W.
116658	α Vir.	19·9	−10 38	1·2	B2	B2	*ns*	−1·9	·024	38, 9	W, t.
118716	ε Cen.	33·5	−52 57	2·6	B1	B2	*ns*	−1·9	·013	12	g.
120307	ν Cen.	43·5	−41 11	3·5	B2	B2	*ss*	−2·7	·006	11	g.
120315	η UMa.	43·6	+49 49	1·9	B3	B3	*ns*	−1·4	·022	− 4	t.
121263	ζ Cen.	49·3	−46 48	3·1	B2*p*	B2	*n*	−1·6	·012	22	g.
122451	β Cen.	56·8	−59 53	0·9	B1	B1	*ss*	−3·2	·015	12, 36	g, t.
127972	η Cen.	14 29·2	−41 43	2·7	B3*p*	B2	*n*	−1·6	·014	19, 13	W, g.
129056	α Lup.	35·3	−46 58	2·9	B2	B2	*s*	−2·3	·009	9	g.
132058	β Lup.	52·0	−42 44	2·8	B2*p*	B2	*ns*	−1·9	·012	18	g.
132200	$\varkappa$ Cen.	52·7	−41 42	3·4	B3	B3	*s*	−1·8	0·009	9	g.

TABLE V.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
H.D.	Name.	1900. R.A.	1900. Dec.	*m.*	Spectrum. H.D.	Spectrum. N.L.O.		M.	π.	Other Parallaxes. π.	Other Parallaxes. Auth.
		h m	° ′						″		
135742	β Lib.	15 11·6	− 9 1	2·7	B8	B7	*ns*	−0·2	0·026		
136298	δ Lup.	14·8	−40 17	3·4	B2	B2	*n*	−1·6	·010		
136504	ε Lup.	15·9	−44 20	3·7	B3	B3	*ns*	−1·4	·009	7	g.
136849	50 Boo.	17·8	+33 17	5·4	B9	B8	*n*	+0·2	·009	10	W.
138690	γ Lup.	28·5	−40 50	3·0	B3	B3	*n*	−1·1	·015	10, 28	g, d.
138749	θ Cr. B.	28·9	+31 42	4·2	B5	B5	*n*	−0·5	·012		
138764	−Lib.	29·0	− 8 51	5·2	B5	B5	*s*	−1·0	·006	6	W.
140873	25 Ser.	40·9	− 1 30	5·4	B8	B6	*ns*	−0·5	·007	7	W.
142669	ϱ Sco.	50·7	−28 55	4·0	B3	B4	*ns*	−1·0	·010	8	g.
143018	π Sco.	52·8	−25 50	3·0	B2	B2	*n*	−1·6	·012	9	g.
143275	δ Sco.	54·4	−22 20	2·5	Bo	B1	*ns*	−2·6	·010	11	g.
144206	υ Her.	59·6	+46 19	4·6	B9	B9	*s*	−0·3	·011		
144217	β Sco.	59·6	−19 32	2·9	B1	B1	*ns*	−2·6	·008	7, 8	W, g.
147165	σ Sco.	16 15·1	−25 21	3·1	B1	B1	*ns*	−2·6	·007	9	g.
148184	χ Oph.	21·2	−18 14	4·9	B3*p*	B3	*nn*	−0·8	·007	9	g.
149212	15 Dra.	28·2	+68 59	5·0	B8*p*	B9	*ns*	0·0	·010	14	W.
149438	τ Sco.	29·7	−28 1	2·9	Bo	B1	*s*	−2·9	·007	10	g.
149757	ζ Oph.	31·7	−10 22	2·7	Bo	Bo	*n*	−3·2	·007		
151890	μ^1 Sco.	16 45·1	−37 53	3·1	B3*p*	B2	*n*	−1·6	·012	8	g.
151985	μ^2 Sco.	45·6	−37 51	3·6	B2	B2	*s*	−2·3	·007	9	g.
152614	ι Oph.	49·3	+10 20	4·3	B8	B8	*ns*	−0·1	·013		
154445	−Oph.	17 0·4	− 0 45	5·6	B3	B3	*s*	−1·8	·003	5	W.
155763	ζ Dra.	8·5	+65 50	3·2	B5	B6	*s*	−0·8	·016	14, 19	W, t.
156633	68 Her.	13·6	+33 12	(4·6)	B3	B3	*n*	−1·1	·007	−21	t.
157056	θ Oph.	15·9	−24 54	3·4	B3	B3	*s*	−1·8	·009		
157246	γ Ara.	17·0	−56 17	3·5	B1	B1	*ns*	−2·6	·006		
158148	−Her.	22·5	+20 10	5·4	B5	B5	*n*	−0·5	·007	7	W.
158408	υ Sco.	24·0	−37 13	2·8	B3	B2	*s*	−2·3	·010		
158427	α Ara.	24·1	−49 48	3·0	B3*p*	B3	*nn*	−0·8	·017	23	g.
158926	λ Sco.	26·8	−37 2	1·7	B2	B2	*s*	−2·3	·016		
159975	μ Oph.	32·4	− 8 3	4·7	B8	B8	*ns*	−0·1	·011		
160578	$\varkappa$ Sco.	35·6	−38 59	2·5	B2	B2	*s*	−2·3	·011	7	g.
160762	ι Her.	36·6	+46 4	3·8	B3	B3	*s*	−1·8	·008		
163472	−Oph.	51·2	+ 0 42	5·7	B3	B3	*ss*	−2·3	·003	4	W.
164284	66 Oph.	55·3	+ 4 23	4·8	B3	B3	*nn*	−0·8	·008		
164353	67 Oph.	55·6	+ 2 56	3·9	B5*p*	B4	*ss*	−2·0	·007	7	W.
164852	96 Her.	58·1	+20 50	5·1	B3	B3	*s*	−1·8	0·004	5, −7	W, t.

TABLE V.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
H.D.	Name.	1900. R.A.	1900. Dec.	*m.*	Spectrum. H.D.	Spectrum. N.L.O.		M.	π.	Other Parallaxes. π.	Other Parallaxes. Auth.
		h m	° ′						″		
165174	−Oph.	59·6	+ 1 55	6·1	B3	B1	*n*	−2·3	0·002		
166182	102 Her.	18 4·4	+20 48	4·3	B3	B3	*ss*	2·3	·005		
166937	μ Sgr.	7·8	−21 5	4·0	B8*p*	B7	*s*	−0·6	·012		
167965	−Lyr.	12·5	+42 8	5·4	B5	B5	*n*	0·5	·007	8	W.
169467	α Tel.	19·6	−46 1	3·8	B3	B3	*ns*	−1·4	·009	14	g.
171301	−Lyr.	29·0	+30 29	5·4	B8	B8	*ns*	−0·1	·008	8	W.
172044	−Lyr.	32·9	+33 23	5·5	B8	B9	*ns*	−0·0	·008	7	W.
173300	φ Sgr.	39·4	−27 6	3·3	B8	B8	*ns*	−0·1	·021		
174[illegible]9	−Lyr.	44·2	+31 39	5·8	B3	B4	*ns*	−1·0	·004		
174585	8 Lyr.	46·1	+32 42	5·8	B2	B3	*ns*	−1·4	·004	5	W.
174933	112 Her	48·0	+21 18	5·3	B9	B9	*s*	0·3	·008		
175191	σ Sgr.	49·1	−26 25	2·1	B3	B3	*ns*	1·4	·020		
175426	δ^1 Lyr.	50·2	+36 51	5·5	B3	B3	*ns*	−1·4	·004	5	W.
175869	64 Ser.	18 52·3	+ 2 26	5·7	B8	B9	*n*	+0·3	·008	9	W.
176871	−Lyr.	57·2	+26 9	5·5	B3	B5	*n*	−0·5	·006		
177003	−Dra.	57·7	+50 24	5·2	B3	B3	*s*	1·8	·004	5	W.
177756	λ Aql.	19 0·9	− 5 2	3·6	B9	B9	*ns*	0·0	·019	38	t.
178125	18 Aql.	2·3	+10 55	5·1	B8	B7	*ns*	−0·2	·009		
178475	ι Lyr.	3·7	+35 57	5·1	B5	B6	*n*	−0·3	·008	8	W.
179761	21 Aql.	8·7	+ 2 7	5·1	B8	B6	*s*	0·8	·007		
180163	η Lyr.	10·4	+38 58	4·5	B3	B3	*ss*	−2·3	·004		
180554	1 Vul.	11·9	+21 13	4·6	B5	B5	*s*	−1·0	·008		
180968	2 Vul.	13·5	+22 51	5·4	B0	B1	*n*	−2·3	·003	6	W.
182255	3 Vul.	18·7	+26 4	4·9	B5	B5	*ns*	−0·7	·008	9	W.
184171	8 Cyg.	28·1	+34 14	4·9	B3	B4	*ns*	−1·0	·007		
184606	9 Vul.	30·2	+19 33	4·9	B8	B7	*n*	0·0	·011		
184915	$\varkappa$ Aql.	31·5	− 7 15	5·0	B0	B1	*n*	−2·3	·004	7	W.
184930	ι Aql.	31·6	− 1 31	4·3	B5	B6	*s*	0·8	·009		
185872	14 Cyg.	36·2	+42 35	5·4	B8	B9	*s*	0·3	·007		
187811	12 Vul.	46·8	+22 21	4·9	B3	B4	*n*	−0·7	·008		
188209	−Cyg.	49·0	+46 47	5·5	B0	B0	*ns*	−3·4	·002	2, 3	W, t.
188665	23 Cyg.	51·2	+57 16	5·0	B5	B5	*s*	−1·0	·006		
188892	22 Cyg.	52·3	+38 13	4·9	B3	B4	*s*	−1·3	·006	6	W.
189178	−Cyg.	53·7	+40 6	5·4	B3	B5	*n*	−0·5	·007	7	W.
189395	−Cyg.	54·7	+30 42	5·4	B8	B9	*ns*	0·0	·008	10	W.
189687	25 Cyg.	56·2	+36 46	5·2	B3	B3	*n*	−1·1	·006		
190993	17 Vul.	20 2·5	+23 19	5·1	B3	B3	*ns*	−1·4	0·005	7	W.

TABLE V.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
H.D.	Name.	1900. R.A.	1900. Dec.	*m.*	Spectrum. H.D.	Spectrum. N.L.O.		M.	π.	Other Parallaxes. π.	Other Parallaxes. Auth.
		h m	° ′						″		
191610	28 Cyg.	5·7	+36 33	4·8	B2*p*	B3	*n*	-1·1	0·007		
192685	—Vul.	11·0	+25 17	4·8	B3	B3	*ns*	-1·4	·006		
192907	$\varkappa$ Cep.	12·3	+77 25	4·4	B9	B9	*s*	-0·3	·012		
193322	—Cyg.	14·6	+40 25	5·8	B2*p*	B2	*n*	-1·6	·003	3. 4	W, t.
193911	25 Vul.	17·7	+24 7	5·4	B8	B8	*ns*	-0·1	·008		
193924	α Pav.	17·7	−57 3	2·1	B3	B3	*ss*	-2·3	·013		
195556	ω^1 Cyg.	27·0	+48 37	4·9	B3	B3	*s*	-1·8	·005		
195810	ε Del.	28·4	+10 58	4·0	B5	B6	*s*	-0·8	·011		
196662	τ Cap.	33·7	−15 18	5·3	B5	B5	*n*	-0·5	·007		
196740	28 Vul.	34·2	+23 46	5·0	B5	B5	*n*	-0·5	·008		
196867	α Del.	35·0	+15 34	3·9	B8	B8	*s*	-0·5	·013		
197511	51 Cyg.	39·1	+49 59	5·4	B3	B3	*ns*	-1·4	·004	5	W.
198478	55 Cyg.	45·5	+45 45	4·9	B2	B2	*ss*	-2·7	·003		
199081	57 Cyg.	49·7	+44 0	4·7	B3	B5	*ns*	-0·7	·008		
200120	59 Cyg.	56·4	+47 8	4·9	B0*p*	B2	*n*	-1·6	·005		
200310	60 Cyg.	57·6	+45 46	5·2	B3	B3	*nn*	-0·8	·006	8	W.
202904	υ Cyg.	21 13·8	+34 29	4·4	B3*p*	B3	*nn*	-0·8	·009	16	t.
204172	69 Cyg.	21·7	+36 14	5·8	B0	B0	*s*	-3·7	·001	2	W.
204403	70 Cyg.	23·2	+36 41	5·2	B3	B4	*ss*	-2·0	·004	6	W.
204770	7 Cep.	25·8	+66 22	5·4	B5	B8	*n*	+0·2	·009	9	W.
205021	β Cep.	27·4	+70 7	3·3	B1	B1	*ss*	-3·2	·005	8	W.
205139	—Cep.	28·3	+60 1	5·5	B0	B1	*s*	-2·9	·002		
205637	ε Cap.	31·5	−19 54	4·7	B5*p*	B4	*s*	-1·3	·006		
206165	9 Cep.	35·2	+61 38	4·9	B2*p*	B2	*ss*	-2·7	·003		
206672	π^1 Cyg.	38·6	+50 44	4·8	B3	B3	*s*	-1·8	·005		
207330	81 Cyg.	43·1	+48 51	4·3	B3	B3	*ss*	-2·3	·005		
208057	16 Peg.	48·5	+25 27	5·1	B3	B4	*s*	-1·3	·005	7	W.
209008	18 Peg.	55·1	+6 14	6·0	B3	B5	*s*	-1·0	·004	4	W.
209409	o Aqr.	58·1	−2 38	4·7	B5*p*	B8	*n*	+0·2	·013		
209481	14 Cep.	58·7	+57 31	5·5	B0	B2	*n*	-1·6	·004		
209819	ι Aqr.	22 1·0	−14 21	4·4	B8	B8	*n*	+0·2	·015		
210129	25 Peg.	3·1	+21 13	5·7	B8	B6	*ns*	-0·5	·006		
212097	32 Peg.	16·7	+27 50	4·9	B8	B8	*s*	-0·5	·008		
212120	2 Lac.	16·9	+46 2	4·7	B5	B5	*ss*	-1·6	·006	37	t.
212571	π Aqr.	20·2	+0 52	4·6	B1*p*	B2	*n*	-1·6	·006		
213420	6 Lac.	26·1	+42 36	4·5	B3	B3	*ss*	-2·3	·004	7	W.
213998	η Aqr.	30·2	−0 38	4·1	B8	B8	*n*	+0·2	·017		
214923	ζ Peg.	36·4	+10 19	3·6	B8	B8	*ns*	-0·1	·018	− 2	t.
214993	12 Lac.	37·0	+39 43	5·2	B2	B2	*ns*	-1·9	·004	4	W.
216200	14 Lac.	45·8	+41 26	5·8	B5	B5	*ns*	-0·7	0·005		

TABLE V. *continued.*

1. H.D.	2. Name.	3. 1900. R.A. h m	4. 1900. Dec. ° ′	5. *m.*	6. Spectrum. H.D.	7. Spectrum. N.L.O.	8. Spectrum. N.L.O.	9. M.	10. *π.*	11. Other Parallaxes. *π.*	12. Other Parallaxes. Auth.
216916	16 Lac.	51·9	+41 4	5·5	B3	B3	*ss*	−2·3	0·003		
217891	*β* Psc.	58·8	+ 3 17	4·6	B5*p*	B5	*nn*	−0·3	·010		
218376	1 Cas.	23 2·4	+58 53	4·9	B1	B1	*s*	−2·9	·003		
218700	58 Peg.	5·0	+ 9 17	5·3	B8	B9	*ns*	0·0	·009	10	W.
219688	ψ^2 Aqr.	12·7	− 9 44	4·6	B5	B5	*n*	−0·5	·010		
220222	64 Peg.	17·0	+31 16	5·4	B8	B5	*s*	−1·0	·005		
220885	13 And.	22·3	+42 21	5·7	B9	B9	*ns*	0·0	·007		
221253	−Cas.	25·4	+58 0	4·9	B3	B3	*ns*	−1·4	·005		
222173	*ι* And.	33·2	+42 43	4·3	B8	B8	*ns*	−0·1	·013		
222304	18 And.	34·3	+49 55	5·3	B9	B9	*ns*	0·0	·009	11	W.
224151	−Cas.	50·5	+56 53	6·1	B0	B1	*s*	−2·9	·002	2	W.
224544	−And.	53·7	+31 48	6·4	B5	B6	*ns*	−0·5	·004		
224572	*σ* Cas.	53·9	+55 12	4·9	B2	B2	*ns*	−1·9	·004		
224926	29 Psc.	56·7	− 3 35	5·2	B8	B7	*s*	−0·6	0·007		

IV. *Comparison of Results with Other Values.*

(a) *Spectroscopic Parallaxes.*

The correlation between line sharpness and absolute magnitude has been utilised by several observers for the purpose of determining spectroscopic parallaxes, notably by Adams and Joy, Schalén (in the papers previously mentioned), and Abetti.* It is therefore of interest to compare the results given in Table V. with those obtained by the foregoing investigators. First, with regard to the relation between the different line-character groups, it may be remarked that those of Mount Wilson and Abetti are more directly comparable with the groups herein described than are those of Schalén, since the latter ignores the effect of spectral type and deduces mean luminosities for different degrees of line sharpness irrespective of the type of spectrum concerned. A comparison of the various classifications is given in Table VI., which shows the frequencies of the classes of other observers which occur in the five classes used in this paper.

TABLE VI.

Frequency Distribution of Line-Character Groups.

N.L.O.	M.W. *s.*	M.W. *n.*	Abetti. *s.*	Abetti. *sn.*	Abetti. *n.*	Schalén. *σσ.*	Schalén. *σσ−.*	Schalén. *σ+.*	Schalén. *σ*
ss	7	3	..	..	..	3	2	1	
s	15	9	2	3	1	3	3	4	
ns	13	38	3	1	7	..	2	10	2
n	1	24	..	..	..	1	3	4	
nn	..	1	..	..	..	..	3	..	

* *Pub. della R. Univ. di Firenze*, fasc. 41 and 42.

It is evident that there is a definite relation between the present character groups and those of Adams and Joy. In the *ss* and *s* groups, 22 out of 34 stars are classed as sharp at Mount Wilson, whereas in the *n* and *nn* groups 25 out of 26 are classed as nebulous. The *ns* group appears to be more nearly allied to the nebulous group of Mount Wilson than to the sharp group, but it is evidently intermediate between the two. A similar result appears to hold in relation to the groups of Abetti, so far as the scanty material available can be interpreted. With regard to the groups of Schalén, however, no definite relation appears to exist. This is scarcely surprising, since Schalén was unable to trace any resemblance between his groups and those of Adams and Joy (*loc. cit.*, p. 27). This lack of resemblance may possibly be attributed to the very much smaller dispersion used by Schalén, causing the wings surrounding the hydrogen lines to be more prominent. It has frequently been noticed that spectral lines occur which are sharp and narrow, but are superimposed on a comparatively broad, hazy, though much fainter band. According to the instrument employed, such a line might be described as either sharp or nebulous.

With regard to spectral type, the classification adopted here agrees with the Harvard classification more closely than do those of Adams and Joy or Abetti. The former, in the stars common to the two lists, show a mean divergence from Harvard of about half a subdivision; the divergence of the latter is nearly one whole subdivision; in the case of the present classification the mean difference is about one-tenth of a subdivision. In each case the revised classifications are "later" than Harvard.

The parallaxes given in column 10 of Table V. show systematic differences from those determined by other spectroscopic methods. These differences amount to 0″·0015 for Mount Wilson, 0″·0036 for Abetti, and 0″·0015 for Schalén. In each case the present parallaxes are smaller than the others; that is to say, the stars are given brighter absolute magnitudes. No attempt has been made to adjust these results by the application of a systematic correction, since even in their present form they are fainter than would be required either by parallactic motions or by trigonometric parallaxes, and are only slightly brighter than the mean from moving cluster data. Apart from the above systematic differences, the agreement with the three other methods is good, except in the case of Schalén's $\sigma +$ group of stars.

The method previously adopted at this observatory (based on relative line intensities) also affords an interesting comparison with the present results. The agreement between the two methods is good, the average difference of parallax (irrespective of sign) being 0″·001. There is a slight systematic difference amounting to 0″·0005 in the mean, corresponding to a difference of 0·1 of a magnitude in luminosity (the earlier values being the brighter).

(b) *Trigonometric Parallaxes.*

A direct comparison with individual trigonometric parallaxes would not be of very great value, owing to the comparatively large

accidental errors and the occurrence of negative values. The paper previously mentioned, by the Astronomer Royal, gives corrected values of trigonometric parallaxes which are more suitable for comparison with the spectroscopic results, and all stars in this list for which such corrected values are available are given in Table VII. In this table, after the number in Schlesinger's Catalogue and name of the star, there is given the observed trigonometric parallax, Dyson's two corrected values (the first, D1, based on corrections given by considering all the stars in Schlesinger's Catalogue; the second, D2, based on corrections from B-type stars alone), the spectroscopic parallax, E, taken from Table V., column 10, and the differences between the spectroscopic and corrected trigonometric parallaxes. These differences are given in units of 0″·001.

TABLE VII.

Comparison with Trigonometric Parallaxes.

Schles.	Star.	Trig. Parallaxes. Obs.	D1.	D2.	E.	E–D1.	E–D2.
		″	″	″	″		
28	ζ Cas.	−0·021	0·005	0·003	0·006	+ 1	+ 3
120	ε Cas.	+ ·001	·009	·005	·013	+ 4	+ 8
223	34 Per.	+ ·015	·014	·008	·008	− 6	0
237	*o* Per.	+ ·013	·013	·007	·007	− 6	0
245	17 Tau.	+ ·020	·018	·007	·014	− 4	+ 7
257	η Tau.	+ ·007	·010	·005	·023	+13	+18
296	48 Per.	+ ·015	·014	·008	·013	− 1	+ 5
405	3 Ori.	+ ·000	·009	·005	·006	− 3	+ 1
408	8 Ori.	+ ·003	·010	·005	·006	− 4	+ 1
446	γ Ori.	+ ·019	·017	·007	·013	− 4	+ 6
449	ψ Ori.	− ·012	·005	·003	·005	0	+ 2
459	δ Ori.	+ ·009	·010	·004	·006	− 4	+ 2
464	ε Ori.	+ ·005	·010	·005	·008	− 2	+ 3
469	σ Ori.	− ·009	·006	·004	·005	− 1	+ 1
470	126 Tau.	+ ·007	·010	·005	·006	− 4	+ 1
931	χ Dra.	− ·003	·006	·005	·013	+ 7	+ 8
947	β Cru.	+ ·008	·010	·005	·012	+ 2	+ 7
982	α Vir.	+ ·009	·010	·005	·024	+14	+19
1010	η UMa.	− ·004	·006	·005	·022	+16	+17
1316	96 Her.	− ·007	·006	·004	·004	− 2	0
1514	Boss 5083	+ ·003	·010	·005	·002	− 8	− 3
1563	Boss 5210	+ ·004	·010	·005	·003	− 7	− 2
1580	ε Del.	+ ·001	·009	·005	·011	+ 2	+ 6
1667	*v* Cyg.	+ ·016	·015	·007	·009	− 6	+ 2
1689	β Cep.	+ ·007	·010	·005	·005	− 5	0
1861	Boss 6142	+ ·029	·025	·012	·002	−23	−10

It will be noticed that the spectroscopic results are intermediate between the two values D1 and D2 of the corrected trigonometric parallaxes. The mean value of the difference E − D1 is − 0″·0012 and of E − D2 is + 0″·0039. If the four stars for which the difference exceeds 0″·010, and which are possibly exceptional in some way, are omitted, the above mean differences become − 0″·0020 and + 0″·0022 respectively, so that the spectroscopic results lie almost exactly midway between the two values D1 and D2. The individual differences are satisfactorily small, considering the fact that the parallaxes in Schlesinger's Catalogue are not homogeneous in nature and Dyson's method is not rigorously applicable to them.

(c) *Moving Cluster Parallaxes.*

Group motion parallaxes were given considerable weight in the formation of the standard curves, and it is therefore to be expected that the final results will, in the mean, agree fairly well with the cluster parallaxes. It is, however, of interest to compare the results for the four main clusters separately, and the mean parallaxes from spectroscopic and group motion sources are accordingly given in Table VIII. for these four clusters, as well as for the mean of all taken together.

Table VIII.

Comparison with Group Motion Parallaxes.

Cluster : Number of Stars :	Pleiades. 5.	Perseus. 16.	Orion. 14.	Sco.–Cen. 29.	All Groups. 64.
Mean π (spectr.)	0″·0138	0″·0085	0″·0067	0″·0113	0″·0099
,, π (group)	0″·0138	0″·0100	0″·0059	0″·0119	0″·0104
,, M (spectr.)	−0·45	−0·75	−2·56	−1·79	−1·61
,, M (group)	−0·45	−0·23	−2·89	−1·65	−1·47
$\Delta\pi$	±0″·0000	+0″·0015	−0″·0008	+0″·0006	+0″·0005
ΔM	0·00	+0·52	−0·33	+0·14	+0·14

The mean differences $\Delta\pi$ and ΔM are the group values *minus* spectroscopic. It will be seen that the Perseus cluster gives the least satisfactory agreement. There is a systematic difference of 0″·0015 for the stars considered, corresponding to a difference of half a magnitude in luminosity. In the Orion cluster the difference is about half the value for Perseus, but of opposite sign. The remaining groups, as well as all the groups taken together, agree well in the mean of their spectroscopic and cluster parallaxes. In the case of the Pleiades the agreement is exact to the fourth decimal place.

Summary.

Owing to the failure, in certain cases, of the spectroscopic method hitherto used for determining parallaxes of B-type stars at the Norman Lockyer Observatory, the method of Adams and Joy has been examined.

Criticisms of the Mount Wilson method are dealt with and slight modifications of it adopted to suit the conditions at this observatory.

The spectra of the B-type stars taken at Sidmouth were first reclassified according to a scheme approaching as nearly as possible to the Harvard system. They were then further divided into five groups according to the sharpness of their spectral lines.

The existence of a relation between luminosity and line character is demonstrated, by means of data from parallactic motion, cluster parallaxes, trigonometric parallaxes, and other sources.

Mean absolute magnitudes are determined for the above five groups in each subdivision of spectral type, and these mean values are finally used to obtain the luminosities and parallaxes of 300 B-type stars (Table V.).

The results are compared with parallaxes from other sources.

Spectroscopic Parallaxes of 300 *Stars of Spectral Class A0–A5.*

By H. C. Woods, Ph.D., B.Sc.

(*Communicated by the Director of the Norman Lockyer Observatory.*)

The stars of Harvard Spectral Class A present a difficult problem in the determination of accurate spectroscopic parallaxes, since the methods of individual line intensities applied to the later classes have not proved reliable in these stars. However, it has been shown * that a good approximation is obtainable by a simple classification into subdivisions of spectral class, together with a second classification which distinguishes the absolutely bright stars from the fainter ones in each subdivision.

A large number of negatives of Class A spectra, taken with the McClean 12-inch Prismatic Camera, being available at the Norman Lockyer Observatory, it was decided to apply a method of this kind to them. The negatives used were of stars classified in the Henry Draper Catalogue as A0–A5, this range being chosen, since it seemed that the general intensity of the metallic lines showed the most uniform progression within these subdivisions and the character of the hydrogen lines could be used as a second basis of classification.

Trigonometric and group motion parallaxes were used as standards, and these were taken from Schlesinger's Catalogue of Parallaxes, 1924. A preliminary solution for the mean absolute magnitudes of groups of stars showed plainly the progression of absolute magnitude both with line character and with spectral class, but the number of standard stars in each group being necessarily small, the deviations of the individual groups rendered the exact relation somewhat obscure. Since these deviations seemed to be largely due to the presence in some of the groups of a preponderance of very small trigonometric parallaxes, it was decided to use the method given by the Astronomer Royal in a recent

* Adams and Joy, *Ap. J.*, **56**. 242.

paper * in order to obtain more reliable means in each of the groups. The number of parallaxes was rather small for the determination of the corrections from these parallaxes alone, and it was thought better to use the corrections given in the paper for the whole of Schlesinger's Catalogue. These corrections made little difference to the deduced relations, but considerably improved the individual results for the groups.

Method of Classification.

The spectra were classified in six groups, A0–A5, a few stars in group A0 being finally assigned to B9 on account of the presence faintly of the helium lines, λλ4026, 4471. Several others were rejected as belonging to a later subdivision than A5. The *a*-stars with spectra similar to that of α Cygni were omitted.

The spectra do not show the metallic lines sufficiently clearly for a detailed examination of their relative intensity or character to be undertaken, but their intensity as a whole could easily be estimated. In class A0 the lines K and λ4481 are the only ones invariably seen, but other lines are visible very faintly in good negatives. The K-line was not used in the classification on account of the poor focus in this region on most of the negatives. The character of the hydrogen lines was estimated and the stars were arranged in seven grades on this basis, each spectral subdivision being spread over about five grades, tending to the sharper classification in the later types.

In each spectral subdivision the standard stars were formed into three groups according to the character of the hydrogen lines. The groups were chosen so that nearly half the stars in each subdivision fell in the middle groups, corresponding to the average line character of the type. The mean absolute magnitude of each of the eighteen groups was computed from the mean parallax, and the central group in each subdivision was used to determine the variation of absolute magnitude with spectral class. The sharp and nebulous groups then gave the rate of variation with line grade, the law being assumed to be a linear one in both cases.

The absolute magnitudes adopted for each spectral subdivision and line grade are given in the following table. The hydrogen-line grades vary in the order shown from the sharpest (*s*) to the most nebulous (*nn*).

Table I.

	s.	*ns+.*	*ns.*	*+ns.*	*n.*	*n+.*	*nn.*
B9	−0·3	−0·2	−0·1	0·1	0·3	0·5	0·8
A0	−0·1	0·1	0·2	0·4	0·6	0·8	1·1
A1	0·2	0·4	0·6	0·8	1·0	1·2	1·4
A2	0·5	0·7	0·9	1·1	1·3	1·5	1·7
A3	0·8	1·0	1·2	1·4	1·6	1·8	2·0
A4	1·1	1·3	1·5	1·7	1·9	2·1	2·3
A5	1·4	1·7	1·9	2·1	2·2	2·4	2·6

* *Monthly Notices, R.A.S.*, **86**, 686.

Binaries.

A number of stars in the list are known spectroscopic binaries, and since the apparent magnitude is the combined magnitude of the two components, some correction should be made in cases where it is known that the secondary component contributes an important part of the light. Accordingly in computing the mean absolute magnitude for the different groups a correction of $+ 0^{m}\cdot5$ was applied to the apparent magnitudes of all stars mentioned in *Lick Obs. Bull.* 355 as spectroscopic binaries showing two spectra. Subsequent comparison of the uncorrected spectroscopic parallaxes for 15 of these stars with the trigonometric and group parallaxes showed that they could be brought into agreement by multiplying by a factor 0·75, which corresponds to a correction to the apparent magnitude $+ 0^{m}\cdot6$. The spectroscopic parallaxes of all such stars have therefore been entered in Table III. with this correction applied, and are placed in brackets. In the case of close visual binaries the spectrum observed is assumed to be that of the brighter star.

Peculiar Spectra.

Several stars with peculiar spectra were omitted from the standard stars used to determine the means for the various groups. Eight of them with trigonometric or group parallaxes available were selected as showing peculiarities of a marked kind, the type of peculiarity being, however, by no means uniform. The spectroscopic parallaxes were greater than the trigonometric and group parallaxes in the ratio of 3 to 2, which corresponds to a difference of $1^{m}\cdot0$ by which the spectroscopic absolute magnitudes are too faint. The number of stars having each kind of peculiarity is far too small for any detailed investigation of their effect on absolute magnitude, but since a feature which seems common to all is the extreme faintness of the K-line for the spectral subdivision to which the star would be assigned on account of the other lines in its spectrum, it is possible that the intensity of this line might be taken into account as a third factor in determining absolute magnitudes.

The average spectroscopic parallax for these 8 stars is $0''\cdot031$, and the corresponding trigonometric and group parallax is $0''\cdot020$, with an average deviation for individual stars of $\pm\ 0''\cdot014$. If the absolute magnitudes are taken $1^{m}\cdot0$ fainter, these are brought into agreement and the average deviation becomes $\pm 0''\cdot008$.

No correction has been applied on account of peculiarity, but the following is a list of those stars which are decidedly abnormal :—

P.G.C. 10	α And.	P.G.C. 3371	α^2 CVn.
,, 550	ι Cas.	,, 3506	78 Vir.
,, 1411	o Aur.	,, 4112	φ Her.
,, 1482	θ Aur.	,, 4284	52 Her.
,, 3363	ε UMa.	,, 6031	$\varkappa$ Psc.

Comparison of Results.

There are 117 stars in the list for which good determinations of trigonometric or group motion parallaxes are available, 107 of these having trigonometric parallaxes, and 23 group motion. The comparison of these with the spectroscopic parallaxes is shown in Table II. A comparison is also made for 26 stars from Jackson and Furner's * lists of hypothetical parallaxes. A satisfactory agreement is shown with four other published lists of spectroscopic parallaxes.

TABLE II.

No. of Stars.	N.L.O. Spect. π.			Av. Deviation.
	″		″	
107	·0322	Trig. π	·0312	±·0125
23	·0455	Group π	·0450	±·0078
26	·0221	Hypoth. π	·0269	±·0125
143	·0284	Adams and Joy †	·0298	±·0055
124	·0256	A. V. Douglas ‡	·0280	±·0065
62	·0252	Abetti (1) §	·0247	±·0045
85	·0224	,, (2)	·0237	±·0048

Description of Table of Spectroscopic Parallaxes.

In Table III. are given the details of the classification of 300 stars, with the resulting spectroscopic absolute magnitudes and parallaxes. The columns are arranged as follows :—

1, 2. Number of star in Boss's Preliminary General Catalogue and name.

3. Apparent magnitude from Revised Harvard Photometry (H.A. 50).

4, 5. Estimated Spectral Class, and character of hydrogen lines.

6, 7. Spectroscopic absolute magnitude and parallax. Parallaxes of spectroscopic binaries corrected and placed in brackets.

8. Trigonometrical parallaxes from Schlesinger's Catalogue of Parallaxes, 1924.

9. Group parallaxes from Schlesinger's Catalogue (*g*), and hypothetical parallaxes by Jackson and Furner (*h*).

* *Monthly Notices, R.A.S.*, **81**, 2 ; *ibid.*, **83**, 4.
† *Ap. J.*, **56**, 242.
‡ *Ap. J.*, **64**, 262.
§ *Publ. Arcetri.*, fasc. 41 ; *ibid.*, fasc. 42.

TABLE III.

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π, etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
10	α And.	2·2	A2	*ns*+	0·7	·051		
43	θ And.	4·4	A3·5	*ns*	1·3	·026		
50	σ And.	4·5	A1·5	*ns*	0·7	(·013)	·014	
82	..	5·2	A2	*ns*+	0·7	·013		
119	..	5·8	A5	*s*	1·4	·013		
145	π Cas.	5·0	A5·5	*ns*+	1·9	(·015)		
150	21 Cas.	5·6	A3·5	*ns*+	1·2	·013		
195	66 Psc.	5·8	A1	*n*+	1·2	·012	·010	·016 *h.*
203	μ And.	3·9	A3	*s*	0·8	·024	·033	
246	41 And.	5·2	A4·5	*ns*+	1·5	·018		
264	θ Cas.	4·5	A4	*ns*	1·5	·025	·007	
282	ζ Psc.	5·5	A5	*n*+	2·4	·023	·022	
300	υ Psc.	4·7	A2·5	*ns*+	0·9	·016	·013	
314	δ Cas.	2·8	A4	+*ns*	1·7	·060		
371	42 Cas.	5·3	A0	*n*	0·6	·011		
422	γ Ari.	4·8	A0	*nn*	1·1	·018		
428	β Ari.	2·7	A4	*ns*+	1·3	·053	·064	
440	..	5·2	A0	*nn*	1·1	·015		
446	48 Cas.	4·6	A4·5	*s*	1·2	·022	·023	·034 *h.*
449	50 Cas.	4·1	A4	*ns*+	1·3	·027		
463	α^2 Psc.	4·3	A3	*ns*+	1·0	·022	..	·042 *h.*
466	ε Tri	5·4	A3	+*ns*	1·4	·016		
480	58 And.	4·8	A3·5	*n*	1·7	·025		
482	β Tri.	3·1	A4	*s*	1·1	(·030)	·014	
508	7 Tri.	5·3	A0	*n*	0·6	·011		
517	γ Tri.	4·1	B9	*nn*	0·8	·020		
522	62 And.	5·1	A0	*n*+	0·8	(·010)		
526	10 Tri.	5·3	A2	*ns*+	0·7	·012		
550	ι Cas.	4·6	A5	+*ns*	2·1	·032	..	·025 *h.*
551	ϱ Cet.	4·9	A0	*n*	0·6	·014		
560	ξ Cet.	4·3	B9	*ns*	−0·1	·012	·021	
606	33 Ari.	5·4	A3	*ns*	1·2	·014	−·003	
622	γ Cet.	3·6	A2	+*ns*	1·1	·032	·038	
666	..	5·3	A1	*n*+	1·2	·015		
668	π Per.	4·6	A2	*n*	1·3	·022		
674	ε Ari.	5·2	A3	*ns*+	1·0	·014	·010	·020 *h.*
677	..	5·2	A3	+*ns*	1·4	·017		
700	ϱ^3 Eri.	5·4	A3	*ns*	1·2	·014		

TABLE III. *continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π, etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
705	..	4·9	B9	*nn*	0·8	·016		
730	ζ Ari.	5·0	A1	*n*	1·0	·016	·016	
791	..	5·0	A1	*ns*	0·6	·013		
850	..	5·4	A3	*ns*	1·2	(·010)		
932	ν Tau.	3·9	A2	*ns*+	0·7	·023	·013	
938	λ Per.	4·3	A0	*n*	0·6	·018		
974	..	5·1	A1	*n*	1·0	·015		
986	b^1 Per.	4·6	A2	+*ns*	1·1	(·015)		
1022	64 Tau.	4·8	A4	*ns*	1·5	·022	..	·028 *g*.
1026	$\varkappa^1$ Tau.	4·4	A4	*ns*	1·5	·026	..	·024 *g*.
1029	68 Tau.	4·2	A4	*ns*	1·5	·032	..	·025 *g*.
1033	υ Tau.	4·4	A4	*ns*+	1·3	(·018)	..	·026 *g*.
1034	71 Tau.	4·6	A4	*ns*+	1·3	·022	−·015	·027 *g*.
1046	θ^2 Tau.	3·6	A5	*ns*+	1·7	(·031)	·028	·025 *g*.
1054	..	4·8	A4·5	*ns*+	1·5	·022	..	·028 *g*.
1067	ϱ Tau.	4·8	A5·5	*ns*+	1·9	·022	·023	·025 *g*.
1076	88 Tau.	4·4	A5	*ns*+	1·7	(·022)		
1087	90 Tau.	4·3	A5	*ns*+	1·7	·030	..	·027 *g*.
1089	σ^1 Tau.	5·2	A5	*ns*+	1·7	·020	(·057)	
1090	σ^2 Tau.	4·9	A4·5	*ns*+	1·5	(·016)	..	·022 *g*.
1095	..	5·0	A3	*ns*+	1·0	·016		
1117	4 Cam.	5·4	A5	*s*	1·4	·016	·012	
1141	2 Ori.	4·4	A0	*nn*	1·1	·022		
1161	7 Cam.	4·4	A3	*ns*+	1·0	·021		
1163	7 Ori.	4·7	A0	+*ns*	0·4	·014		
1194	ι Tau.	4·7	A5	+*ns*	2·1	·030		
1214	106 Tau.	5·3	A5	*ns*	1·9	·021		
1220	β Eri.	2·9	A3	+*ns*	1·4	·051	·052	
1236	μ Aur.	4·8	A5	*s*	1·4	·021	(·068)	
1241	μ Lep.	3·3	A1	*ns*+	0·4	·026		
1268	19 Aur.	5·2	A3	*ns*+	1·0	·014		
1320	118 Tau.	5·4	B9	*nn*	0·8	·011	..	·022 *h*.
1392	49 Aur.	4·9	A3	*n*+	1·8	(·018)		
1411	o Aur.	5·5	A2·5	*n*	1·5	·015	·014	
1432	ζ Lep.	3·7	A3	*nn*	2·0	·046		
1453	ξ Aur.	4·9	A2	*nn*	1·7	·021	(−·014)	
1478	β Aur.	2·1	A2	*ns*+	0·7	(·040)	·034	·025 *g*.
1482	θ Aur.	2·7	A1·5	*s*	0·3	·034	·016	·055 *h*.

TABLE III.—*continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π, etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
1501	μ Ori.	4·2	A5	*s*	1·4	·028	·043	
1522	θ Lep.	4·7	A0	*nn*	1·1	·019		
1575	2 Lyn.	4·4	A3	*ns*+	1·0	·021	·034	
1611	8 Mon.	4·5	A4	*ns*+	1·3	·023	·016	
1668	49 Aur.	5·0	A0	*n*	0·6	·013		
1690	γ Gem.	1·9	A2·5	*ns*+	0·9	·063	·043	
1714	26 Gem.	5·1	A1	*ns*	0·6	·013		
1716	12 Lyn.	4·9	A2	+*ns*	1·1	·017	..	·024 *h.*
1732	α CMa.	−1·6	A3	*ns*	1·2	·364	·371	·380 *g.*
1759	36 Gem.	5·2	A1	*ns*	0·6	·012		
1763	θ Gem.	3·6	A3	*ns*+	1·0	·030	·019	
1786	16 Lyn.	4·8	A2	*nn*	1·7	·022		
1853	δ Mon.	4·1	A0	*n*	0·6	·020		
1886	λ Gem.	3·6	A3	*ns*+	1·0	·030	·038	
1928	21 Lyn.	4·5	A1	*nn*	1·4	·024		
1956	64 Gem.	5·0	A3	+*ns*	1·4	·019		
1977	68 Gem.	5·1	A1	*nn*	1·4	·018	·008	
2078	φ Gem.	5·0	A4	+*ns*	1·7	(·016)		
2138	8 Cnc.	5·1	A1·5	*ns*	0·8	·013		
2145	27 Lyn.	4·9	A1	*n*	1·0	·017	−·017	
2237	30 Mon.	4·0	A1	*n*+	1·2	·028	·008	
2295	δ Hya.	4·2	B9	*nn*	0·8	·020		
2327	γ Cnc.	4·7	A2	*ns*	0·9	·017	·006	
2361	ϱ Hya.	4·4	A0	*ns*	0·2	·015	·004	
2400	o^1 Cnc.	5·2	A5	*s*	1·4	·017		
2404	ι UMa.	3·1	A4	*s*	1·1	·040	(·070)	·102 *h.*
2424	$\varkappa$ UMa.	3·7	A1	*ns*	0·6	·024		
2476	18 UMa.	4·9	A3	*ns*	1·2	·018		
2479	θ Hya.	3·8	A0	+*ns*	0·4	·021	·017	
2495	38 Lyn.	3·8	A3	*nn*	2·0	·044	·032	·014 *h.*
2559	32 Hya.	4·5	A4	*ns*+	1·3	·023		
2565	26 UMa.	4·7	A1	*nn*	1·4	·022		
2584	42 Lyn.	5·2	A4	*s*	1·1	·015		
2637	φ UMa.	5·0 5·6	A2	*ns*+	0·7	·014	..	·011 *h.*
2642	22 Leo.	5·3	A5	*s*	1·4	·017	·035	
2672	ν Leo.	5·2	A1	*ns*	0·6	·012		
2692	21 LMi.	4·5	A4	*ns*	1·5	·025		

TABLE III.—*continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π, etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
2697	15 Sex.	4·5	A1	*ns*+	0·4	·015		
2729	λ UMa.	3·5	A3	*s*	0·8	·029		
2754	..	4·9	A2	*ns*+	0·7	·015		
2773	..	5·8	A4	*ns*	1·5	·014		
2802	..	4·8	A3	*ns*	1·2	·019	·026	
2852	40 LMi.	5·6	A2	+*ns*	1·1	·013		
2854	41 LMi.	5·0	A0	*nn*	1·1	·017		
2883	53 Leo.	5·3	A1	*nn*	1·4	(·013)		
2900	ω UMa.	4·8	A2	+*ns*	1·1	(·014)		
2909	54 Leo.	4·5	A0·5	+*ns*	0·6	·017	..	·025 *h*.
2928	59 Leo.	5·1	A4	*s*	1·1	·016		
2930	β UMa.	2·4	A1	*ns*	0·6	·044	..	·047 *g*.
2932	60 Leo.	4·4	A3	*s*	0·8	·019	·010	
2956	67 Leo.	5·6	A3	*ns*	1·2	·013		
2970	69 Leo.	5·4	A1	*n*+	1·2	·014		
2972	δ Leo.	2·6	A3	+*ns*	1·4	·058	·078	
2974	θ Leo.	3·4	A2·5	*ns*+	0·9	·031	·019	
2982	φ Leo.	4·6	A4	*n*	1·9	·029		
2987	55 UMa.	4·8	A3	*ns*	1·2	(·014)	(·013)	
2990	σ Leo.	4·1	A0	*ns*	0·2	·017		
3005	γ Crt.	4·1	A5	*ns*+	1·7	·033	·030	
3088	ξ Vir.	5·1	A3	*ns*	1·2	·017		
3097	4 Vir.	5·2	A1	+*ns*	0·8	·013		
3101	β Leo.	2·2	A3	*n*+	1·8	·083	·101	
3117	γ UMa.	2·5	A1	*n*	1·0	·050	·004	·041 *g*.
3135	7 Vir.	5·2	A1	*ns*	0·6	·012		
3139	π Vir.	4·6	A3	*ns*	1·2	·021		
3190	δ UMa.	3·4	A2	*nn*	1·7	·042	..	·045 *g*.
3192	6 Com.	5·1	A1	*n*	1·0	·015		
3210	η Vir.	4·0	A3	*ns*	1·2	(·021)		
3231	13 Com.	5·1	A4	*ns*+	1·3	·017		
3240	14 Com.	5·1	A4	*ns*	1·5	·019		
3244	16 Com.	5·0	A3·5	*ns*+	1·2	·017		
3251	17 Com.	5·4	A4	*s*	1·1	·014		
3260	74 UMa.	5·4	A5	*ns*+	1·7	·018		
3266	21 Com.	5·4	A3	*ns*	1·2	·014		
3277	21 Vir.	5·4	A0	*n*+	0·8	·012		
3283	23 Com.	4·8	A1·5	*ns*	0·7	(·012)		

TABLE III.—*continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π, etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
3309	ϱ Vir.	5·0	B9	*n*	0·3	·010		
3310	31 Vir.	5·5	B9	*n*+	0·5	·009		
3339	30 Com.	5·8	B9	*n*	0·3	·007		
3363	ε UMa.	1·7	A3	*ns*+	1·0	·073	..	·042 *g.*
3371	α^2 CVn.	2·8	A2	*s*	0·5	·035	·015	
3409	θ Vir.	4·4	A2	+*ns*	1·1	·022	·007	
3450	21 CVn.	5·1	A0	+*ns*	0·4	·011		
3474	ζ^1 UMa.	2·4	A1	*n*	1·0	(·040)	·038	·045 *g.*
3480	80 UMa.	4·0	A3	*n*+	1·8	(·027)	·035	·042 *g.*
3494	..	5·4	A0	*nn*	1·1	·014		
3506	78 Vir.	4·9	A4	*n*	1·9	·025	·024	·014 *g.*
3508	ζ Vir.	3·4	A2	*n*	1·3	·038	·038	
3509	81 UMa.	5·5	A1	*n*+	1·2	·014		
3512	24 CVn.	4·6	A3·5	*ns*	1·3	(·016)		
3526	..	{6·3 / 6·3}	A5	*s*	1·4	·010		
3530	82 UMa.	5·3	A3	*nn*	2·0	·022		
3531	1 Boo.	5·7	A3	+*ns*	1·4	·014	..	·028 *h.*
3612	τ Vir.	4·3	A2·5	*ns*	1·1	·023		
3626	α Dra.	3·6	A1	*ns*	0·6	·025		
3639	..	4·9	A0	*n*	0·6	·014		
3654	$\varkappa^2$ Boo.	4·6	A3	*n*	1·6	·025	·025	
3663	..	5·8	A2	*n*+	1·5	·014		
3666	λ Boo.	4·3	A0	+*ns*	0·4	·017	·041	
3667	ι Boo.	4·8	A5	*ns*	1·9	·026		
3672	λ Vir.	4·6	A4	*ns*	1·5	(·018)		
3697	..	5·1	A3	*n*+	1·8	·022		
3744	33 Boo.	5·4	A1	+*ns*	0·8	·012		
3749	π^1 Boo.	4·9	A2	*ns*	0·9	·016		
3752	ζ Boo.	4·4	A2	*n*	1·3	·024	..	·019 *h.*
3772	109 Vir.	3·8	A0	*nn*	1·1	·029		
3825	δ Lib.	4·8	A0	*n*	0·6	·014		
3826	40 Boo.	5·6	A5	*ns*+	1·7	·017		
3883	χ Boo.	5·3	A2	*n*	1·3	·016		
3928	γ UMi	3·1	A3	*ns*+	1·0	·038		
3939	10 Ser.	5·1	A4	+*ns*	1·7	·021		
3949	ν^2 Boo.	5·0	A4	*n*+	2·1	·026		
3960	δ Ser.	4·2	A5	+*ns*	2·1	·038	·016	

TABLE III.—*continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π, etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
3961	α Cr. B.	2·3	A0	*ns*	0·2	·038	·053	·041 *g.*
3994	ι Ser.	4·5	A1	+*ns*	0·8	(·013)	·007	
3998	γ Cr. B.	3·9	A0	*ns*	0·2	(·013)	·022	·029 *h.*
4004	..	5·5	A3	+*ns*	1·4	·015		
4009	β Ser.	3·7	A2	*ns*	0·9	·028	·032	·023 *g.*
4016	μ Ser.	3·6	A1	*nn*	1·4	·036		
4026	ε Ser.	3·8	A4·5	*s*	1·3	·031	·036	
4028	36 Ser.	5·2	A1	*nn*	1·4	·017		
4035	ζ UMi	4·3	A2	*n*+	1·5	·028		
4080	ι Cr. B.	4·9	A1	*ns*	0·6	·014		
4081	π Ser.	4·8	A3	+*ns*	1·4	(·016)		
4112	φ Her.	4·3	A1	+*ns*	0·8	·020	·014	
4182	ω Her.	4·5	A4	*ns*+	1·3	·023		
4189	υ Oph.	4·7	A4	*ns*+	1·3	·021		
4203	λ Oph.	3·8	A2	*ns*	0·9	·026	·000	·038 *h.*
4220	σ Her.	4·3	A1	*n*	1·0	·022		
4232	17 Dra.	5·7	A1	+*ns*	0·8	·010		
4280	47 Her.	5·5	A4	*ns*+	1·3	·014		
4284	52 Her.	4·9	A4	*n*	1·9	·025	·003	·035 *h.*
4285	21 Oph.	5·5	A2	*nn*	1·7	·016	..	·009 *h.*
4300	53 Her.	5·3	A5	*ns*+	1·7	·019		
4328	ε Her.	3·9	A2	*ns*	0·9	·025	·023	
4332	59 Her.	5·3	A4	*ns*+	1·3	·016		
4346	60 Her.	4·9	A3	*ns*	1·2	·018		
4360	η Oph.	{2·2 3·7}	A3	*ns*	1·2	·040	..	·016 *h.*
4376	δ Her.	3·2	A1	*nn*	1·4	·044	·029	
4391	69 Her.	4·8	A1	*nn*	1·4	·021		
4395	ν Ser.	4·3	A1	+*ns*	0·8	·020		
4401	..	5·3	A3	*s*	0·8	·013		
4402	70 Her.	5·1	A1	*n*	1·0	·015		
4419	ϱ Her.	4·5	A2	*ns*	0·9	·019	−·002	·017 *h.*
4428	..	5·2	A5	+*ns*	2·1	·024		
4458	ν^1 Dra.	5·0	A4·5	*s*	1·2	·018	·023	
4459	α Oph.	2·1	A3	*n*+	1·8	(·065)	·049	
4460	ν^2 Dra.	5·0	A5	*s*	1·4	·019	·023	
4462	ξ Ser.	3·6	A5	+*ns*	2·1	·050		
4475	o Ser.	4·4	A2	*ns*	0·9	·020		

TABLE III.—*continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π. etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
4500	γ Oph.	3·7	A1	*nn*	1·4	(·026)		
4511	30 Dra.	5·2	A2	+*ns*	1·1	·015	−·004	
4552	68 Oph.	4·4	A1	*nn*	1·4	·025		
4584	*o* Her.	3·8	A0	*n*+	0·8	·025		
4591	δ UMi.	4·4	A0	*n*	0·6	·017		
4592	101 Her.	5·2	A4	*ns*+	1·3	·017		
4644	107 Her.	5·0	A3	*n*	1·6	·021		
4661	μ Lyr.	5·0	A1·5	*n*	1·2	·017		
4670	φ Dra.	4·2	B9	*ns*	−0·1	(·010)	..	·005 *h.*
4671	39 Dra.	4·8	A2	*nn*	1·7	(·017)	·036	·026 *h.*
4674	γ Sct.	4·7	A2	+*ns*	1·1	·019		
4722	α Lyr.	0·1	A0·5	*ns*+	0·3	·108	·124	
4745	46 Dra.	5·1	A0	*n*	0·6	(·010)		
4761	111 Her.	4·4	A4	*ns*+	1·3	·024	·054	
4775	ν^2 Lyr.	5·2	A2	*ns*	0·9	·014		
4788	50 Dra.	5·4	A1	*n*	1·0	·013		
4846	16 Lyr.	5·1	A3	*ns*	1·2	·017		
4858	ζ Aql.	3·0	A0	*nn*	1·1	·042	·040	
4914	ω Aql.	5·1	A4	*ns*+	1·3	·017	·014	
4948	π Dra.	4·6	A3·5	*s*	1·0	·019	·016	
4965	5 Vul.	5·6	A1	*n*	1·0	·012	(·029)	
4988	ι Cyg.	3·9	A4	+*ns*	1·7	·036	·005	
5048	δ Cyg.	3·0	A0	*n*	0·6	·033	·038	·036 *h.*
5058	ζ Sge.	5·0	A2	*n*	1·3	·018	·024	·030 *h.*
5062	α Aql.	0·9	A3	*n*+	1·8	·152	·204	
5086	13 Vul.	4·5	A0	*ns*	0·2	·014		
5105	ψ Cyg.	4·8	A3	*ns*	1·2	·019	·005	·009 *h.*
5132	15 Vul.	4·7	A5·5	*ns*+	1·9	·028	·030	
5171	θ Aql.	3·4	A0	*ns*	0·2	(·017)	·015	
5182	ϱ Aql.	5·0	A1	*n*	1·0	·016		
5186	o^1 Cyg.	5·0	A3	*ns*+	1·0	·016	·009	
5188	29 Cyg.	5·0	A0	*ns*	0·2	·011		
5191	33 Cyg.	4·3	A3	*n*+	1·8	·032		
5214	ν Cap.	4·8	A2	+*ns*	1·1	·018		
5270	θ Cep.	4·3	A4	*s*	1·1	·023		
5275	η Del.	5·2	A3	*n*	1·6	·021		
5282	ζ Del.	4·7	A4	*ns*+	1·3	·021	·015	
5301	29 Vul.	4·8	A0	*n*+	0·8	·016	·001	

TABLE III.—*continued.*

P.G.C. No.	Star.	App. Mag.	Est. Sp. Cl.	H. Lines.	Sp. Abs. Magn.	Sp. π.	Trig. π.	Group π. etc.
1.	2.	3.	4.	5.	6.	7.	8.	9.
5337	ε Aqr.	3·8	A1	+*ns*	0·8	·025	·022	
5365	56 Cyg.	5·1	A5	*ns*+	1·7	·021	·030	
5393	ν Cyg.	4·0	A0	*n*	0·6	·021	·003	
5417	η Cap.	4·9	A3	+*ns*	1·4	·020		
5427	θ Cap.	4·2	A1	+*ns*	0·8	·021		
5480	α Cep.	2·6	A4	+*ns*	1·7	·066	·083	
5491	β Equ.	5·1	A2	*ns*+	0·7	(·010)	(·082)	
5515	35 Vul.	5·4	A3	+*ns*	1·4	·016		
5517	..	5·3	A4	+*ns*	1·7	·019	·006	
5551	ξ Aqr.	4·8	A4	+*ns*	1·7	·024		
5553	74 Cyg.	5·1	A4	+*ns*	1·7	·021	·018	
5573	76 Cyg.	6·0	A0	*n*	0·6	·008	·009	·013 *g.*
5579	77 Cyg.	5·5	B9	*n*	0·3	(·006)	·001	
5600	δ Cap.	3·0	A5	*ns*	1·9	·060	·114	
5617	14 Peg.	5·0	A1	*nn*	1·4	(·014)	·004	
5703	θ Peg.	3·7	A2	*n*	1·3	(·025)		
5713	..	5·4	A3	*n*	1·6	·017	·018	
5734	..	5·5	A2	*n*	1·3	·014		
5761	γ Aqr.	4·0	A1	+*ns*	0·8	·023	·064	
5803	σ Aqr.	4·9	A1	*n*	1·0	·017		
5813	α Lac.	2·9	A1	*n*	1·0	·042	·034	
5837	9 Lac.	4·8	A5	*ns*	1·9	·026	·020	
5848	30 Cep.	5·2	A1	*n*	1·0	(·010)		
5858	ο Peg.	4·9	A1	+*ns*	0·8	·015		
5904	δ Aqr.	3·5	A3	*n*	1·6	·042		
5910	ϱ Peg.	5·0	A1	*n*	1·0	·016	·002	
5916	α PsA.	1·4	A4	*s*	1·1	·087	·137	
5936	2 And.	5·1	A2	*n*+	1·5	·019	·055	·013 *h.*
5944	α Peg.	2·6	A0	*ns*+	0·1	(·023)	·038	
5997	95 Aqr.	5·2	A0	+*ns*	0·4	·011		
6005	τ Peg.	4·6	A4	*ns*+	1·3	·022	·034	
6031	κ Psc.	4·9	A4	*n*	1·9	·025	·041	
6072	75 Peg.	5·4	A0	*nn*	1·1	·014		
6080	κ And.	4·3	B9	*n*+	0·5	·020	·008	
6084	λ Psc.	4·6	A4·5	*ns*+	1·5	·026	·030	·016 *g.*
6087	105 Aqr.	4·6	A1	*ns*	0·6	·016		
6179	2 Cet.	4·6	A0	*n*	0·6	·016		

Accuracy of Results.

The spectroscopic method of determining parallax depends on the assumption that a complete correlation can be found between one or more spectral characteristics and the absolute magnitude of a star. The limitations as to number and accuracy of the standard parallaxes render a satisfactory proof of such an assumption impossible, but if there is evidence that, when an individual star must be considered apart from the rest, some conspicuous feature of its spectrum will indicate its peculiarity, it should be possible to estimate the greatest deviation to be expected in a case where conspicuous peculiarities are not found. Thus in the present list, those stars which have been selected on account of very strong peculiarities show a deviation amounting to about one magnitude. The great majority of the stars, however, resemble the average of their type much more closely, and it seems a fair conclusion that the deviations among them which can be attributed to errors of the fundamental assumption cannot amount to any large fraction of a magnitude in more than a very few cases.

Errors may also arise from inaccuracy in the classification of the spectra. These principally depend on the quality of the negatives used. Poor negatives will usually lead to the assignment of too nebulous a line grade and too early a spectral type, but only those negatives were used which were of fair to good quality, and the agreement in both classifications where two or more spectra of the same star were available was quite satisfactory. As the negatives were examined in chronological order, and considerable periods elapsed between the negatives of the same star in many cases, it is not likely that serious errors have been introduced in this way.

In conclusion, the author wishes to acknowledge his obligation to the Norman Lockyer Observatory Corporation for the studentship which has enabled him to spend a year at this observatory, and to the Director, Dr. W. J. S. Lockyer, who has generously given every opportunity for the use of the instruments and equipment. He is also greatly indebted to the Chief Assistant, Mr. D. L. Edwards, by whom most of the photographs used in this investigation were taken.

Summary.

Spectroscopic parallaxes have been found for 300 stars of Harvard Spectral Classes A0–A5, using a method of classification with regard to spectral class and character of the hydrogen lines.

The standard parallaxes have all been taken from Schlesinger's Catalogue of Parallaxes, 1924, and have been used with the corrections recently published by the Astronomer Royal, to obtain a better mean parallax for small groups of stars.

Corrections have been applied in the case of certain spectroscopic binary stars, but it has not been possible to make any definite allowance for peculiar spectra.

Comparisons are given with the standard parallaxes and with other lists of spectroscopic parallaxes.

On Liquid Stars and the Liberation of Stellar Energy.

By J. H. Jeans.

1. Since the days of Lane's Law and the Helmholtz contraction hypothesis, stars have been regarded as gaseous structures, so much so that Emden's *Gas Kügeln* (1907) discusses no alternative possibility. Yet Emden's calculations made it clear that observed stellar densities were incompatible with a gaseous state in which the separate units were complete molecules. The question assumed a different aspect in the light of the concept I put forward in 1917,* according to which the atoms in a star's interior were in a high degree of electronic dissociation. If the flying units were electrons and nuclei of diminutive size, it became impossible to say that any observed density was too great to be compatible with the gaseous state.

Throughout this period Poincaré, Darwin, and others were studying the formation of binary stars by fission, on the supposition that stellar matter might be treated as an incompressible liquid. In 1917 † I showed that fission could occur only in stars whose interior state at least approximated to incompressibility. With even the lowest degree of central condensation which could occur in a purely gaseous star fission was proved to be an impossibility,‡ any excess of angular momentum relieving itself by an equatorial ejection of matter of the general type observed in the spiral nebulæ. About one-third of the stars observed in the sky are binaries which have almost certainly been formed by fission, so that these, at least, must have been in something approximating to the liquid state when fission took place. Thus the direct evidence of observational astronomy pronounces in favour of the stars being liquid rather than gaseous structures.

I recently published an investigation § of the mode of liberation of stellar energy based on a mathematical discussion of the dynamical stability of a star. Certain modes of liberation of energy were found to make a star either explosive or unstable, and so can be ruled out. I have since found that the conditions for stability are far more restrictive than I then thought, and the possible modes of generation of energy can be further narrowed down. In particular, the field of possibilities disappears altogether when the stellar matter behaves like a perfect gas. Thus every existing star is kept stable solely by its deviations from the gas laws, and this brings us rapidly to the conception of stars which are well on towards the liquid state.

It is not, of course, to be supposed that a star can approximate to the liquid state throughout ; its outer layers must always be purely gaseous. Nevertheless, the phenomenon of fission appears to demand that an

* Bakerian Lecture, *Phil. Trans*, **218**, p. 209.

† *Problems of Cosmogony*, p. 186.

‡ Milne has claimed to prove (*M.N.*, **83** (1923), p. 141) that a rotating star of high mass would approximate to uniform density. But I consider his proof is invalid, terms being omitted which I believe ought to be retained. A similar criticism has been made by Vogt (*Ast. Nach.*, 5480).

§ *M.N.*, **85** (1925), p. 914.

approximation to the liquid state must extend throughout a large part of the star's *mass*, although not necessarily throughout a large part of its *volume*.

In the present paper the next few sections (§§ 2–11) contain a discussion of the relation between the mode of liberation of a star's energy and its stability, it being found that considerations of stability practically limit us to the "uranium" model for the mode of liberation of energy, and that even then (§ 12) the star can only rest stably in configurations in which there are substantial deviations from the gas laws. We next examine (§§ 13–15) in what actual configurations of a star these deviations will occur. At different temperatures the atoms will of course be of different sizes through being ionised down to different levels. As a star contracts its density increases, but so also does its temperature, with the result that the atoms diminish in size. There is most likely to be an approach to the fluid state in the configurations which occur just before the various rings of electrons are ionised. Detailed calculation suggests that the "main sequence" represents configurations in which the atoms are ionised down to their K-rings and jammed together; the "giant branch" similarly represents L-ring jamming, and the white dwarfs nuclear jamming. The general scheme is shown in fig. 1 (p. 413).

Dynamical Equations.

2. The star under discussion will be supposed always to remain spherical; it may expand or contract, and the temperature of each spherical shell may change. The configuration of the star is settled when T, the temperature, and r, the distance from the centre, are given for each spherical shell. The values of the density ρ, of course, follow at once from the values of r.

In discussing the equilibrium of a star at rest, two equations are necessary: (1) An equation of dynamical equilibrium expressing that the total pressure at any point is just adequate to support the total weight of gas above that point, and (2) an equation of radiative equilibrium, expressing that the generation of energy in any small volume of the star is just adequate to make good the net loss by radiation.

In the same way, when the star is in motion, two equations are necessary to discuss that motion: (1) a dynamical equation expressing that any pressure in excess or defect of the amount requisite for equilibrium produces accelerations in the star, and (2) a radiative equation expressing that any excess in the generation of energy over the net amount radiated away produces an increase in the heat energy of the stellar matter. These equations are easily found to be *

$$\frac{d^2r}{dt^2} = -\frac{1}{\rho}\frac{d}{dr}(p_G + \tfrac{1}{3}aT^4) - \frac{\gamma M_r}{r^2} \quad . \quad . \quad (1)$$

$$\rho C_v \frac{dT}{dt} - (p_G + \tfrac{1}{3}aT^4)\frac{1}{\rho}\frac{d\rho}{dt} = \rho G - \frac{1}{r^2}\frac{d}{dr}(r^2 H) \quad . \quad (2)$$

* *M.N.*, **85** (1925), p. 917.

Here all the symbols have their usual meanings, G being the rate of generation of energy per unit mass, and H being the outward flow of radiation per unit area. We obtain the usual equations of equilibrium for a star at rest on equating the right-hand members to zero.

3. At first we discuss only a special model of stellar structure, or rather, a structure about which special assumptions are made. We return to generality at the end of § 12. For the present we suppose that the star already has such stability that a local disturbance is spread in the form of waves until it is uniformly spread throughout the star. We proceed to consider the stability, or reverse, of a star affected by a disturbance which is uniformly spread throughout the star. To represent this we suppose the star's motion to be such that r changes proportionately throughout the star; the motion at each instant is one of uniform contraction or expansion. We also suppose T to change proportionately throughout the star, so that at every instant it is being uniformly heated or cooled. When these assumptions are made, the changes in any one shell or element of the star are characteristic of those of the whole star.

Let the equilibrium values of T, ρ, G, H . . . for any shell of the star be T_0, ρ_0 . . ., and in the displaced configuration let their values be $T_0 + \delta T_0$, $\rho_0 + \delta\rho_0$. . . . Throughout the motion we must of course have

$$\frac{\rho_0}{\rho} = \left(\frac{r}{r_0}\right)^3.$$

We shall allow for departures from the ordinary gas laws from the outset by supposing that

$$p_G \propto T\rho^{1+s} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (1), the dynamical equation of motion, now assumes the form

$$\frac{d^2r}{dt^2} = -\frac{Tr_0^{1+3s}}{T_0 r^{1+3s}}\left(\frac{1}{\rho}\frac{dp_G}{dr}\right)_0 - \frac{T^4r^2}{T_0^4 r_0^2}\left(\frac{d}{dr}(\tfrac{1}{3}aT^4)\right)_0 - \frac{\gamma M_r}{r_0^2}\left(\frac{r_0}{r}\right)^2.$$

If λ denotes the equilibrium value of the ratio of gas pressure to pressure of radiation, we have

$$\left(\frac{1}{\rho}\frac{dp_G}{dr}\right)_0 = -\frac{\lambda}{\lambda+1}\left(\frac{\gamma M}{r_0^2}\right); \qquad \left(\frac{1}{\rho}\frac{d}{dr}(\tfrac{1}{3}aT^4)\right)_0 = -\frac{1}{\lambda+1}\left(\frac{\gamma M}{r_0^2}\right),$$

and the equation becomes

$$\frac{d^2r}{dt^2} = \frac{\gamma M}{r_0^2}\left[\frac{\lambda}{\lambda+1}\left(\frac{Tr_0^{1+3s}}{T_0 r^{1+3s}}\right) + \frac{1}{\lambda+1}\left(\frac{T^4r^2}{T_0^4 r_0^2}\right) - \frac{r_0^2}{r^2}\right],$$

or, in terms of the small increments δr, δT,

$$\frac{d^2}{dt^2}(\delta r) = \frac{\gamma M}{r_0^2}\left[\frac{\lambda+4}{\lambda+1}\left(\frac{\delta T}{T_0} + \frac{\delta r}{r_0}\right) - \frac{3s\lambda}{\lambda+1}\frac{\delta r}{r_0}\right]. \quad . \quad (4)$$

A displaced configuration will be one of dynamical equilibrium,

although not necessarily of radiative equilibrium, if the right-hand member vanishes. For a perfect gas, $s = 0$ and the condition reduces to

$$\frac{\delta T}{T_0} + \frac{\delta r}{r_0} = 0,$$

which has the well-known integral $rT = r_0T_0$ (Lane's Law). More generally, if s is regarded as being constant over a small range of configurations, the condition that these shall form a series of configurations in dynamical equilibrium is

$$r^{1-3s\lambda/(\lambda+4)}T = r_0^{1-3s\lambda/(\lambda+4)}T_0.$$

4. Let us suppose that the coefficient of opacity is proportional to $\rho T^{-(3+n)}$. If we put $n = \frac{1}{2}$ we obtain the principal factor in Kramer's Law. The second factor, which is usually written $1 + h\nu/RT$, remains approximately constant throughout the whole series of changes of a star, but at any special temperature it may be changing fairly rapidly. We shall absorb the changes in this factor into our assumed factor $T^{-(3+n)}$. The mean value of n over a long range will thus be about $\frac{1}{2}$. Moreover, since $h\nu$ increases as T increases, the value of n can never, even in the most extreme case, be greater than $\frac{3}{2}$. Finally, if a star were composed of entirely similar atoms, there might be considerable fluctuations in the value of n, but in a star composed of a mixture of elements, the steps in the value of $h\nu$ will occur at different temperatures for the different elements, so that the fluctuations in n tend to neutralise one another.

With this law for the coefficient of opacity, the flux of energy per unit area H is proportional to

$$\left(\frac{T^3}{\rho}\right)^2 \frac{\partial T}{\partial r} T^n,$$

whence we readily obtain

$$\frac{1}{r^2}\frac{d}{dr}(r^2H) = \left(\frac{T}{T_0}\right)^{7+n}\left(\frac{r}{r_0}\right)^4\left[\frac{1}{r_0^2}\frac{d}{dr_0}(r_0^2H_0)\right] \quad . \quad . \quad (5)$$

From the equation of equilibrium (*cf.* equation (2)),

$$\frac{1}{r_0^2}\frac{d}{dr_0}(r_0^2H_0) = \rho_0 G_0,$$

so that equation (5) assumes the form

$$\frac{1}{r^2}\frac{d}{dr}(r^2H) = \left(\frac{T}{T_0}\right)^{7+n}\left(\frac{r}{r_0}\right)^7 \rho G_0 \quad . \quad . \quad . \quad (6)$$

Equation (2) may now be written in the form (dividing by ρ and writing p for the total pressure):

$$C_v\frac{dT}{dt} + \frac{3p}{\rho r}\frac{dr}{dt} = G - \left(\frac{T}{T_0}\right)^{7+n}\left(\frac{r}{r_0}\right)^7 G_0,$$

and the new configuration will be in radiative equilibrium only if G/G_0 has such a value that the right-hand member of this equation vanishes.

For the present we suppose that G, the rate of generation of energy, depends on both T and ρ, being proportional to $\rho^\alpha T^\beta$, so that

$$\frac{G}{G_0} = 1 + \alpha\frac{\delta\rho}{\rho_0} + \beta\frac{\delta T}{T_0} = 1 - 3\alpha\frac{\delta r}{r_0} + \beta\frac{\delta T}{T_0}.$$

Equation (2), the equation of radiative equilibrium, now assumes the form

$$C_v\frac{d}{dt}(\delta T) + \frac{3p}{\rho r}\frac{d}{dt}(\delta r) = G_0\left(-3\alpha\frac{\delta r}{r_0} + \beta\frac{\delta T}{T_0} - \frac{(7+n)\delta T}{T_0} - \frac{7\delta r}{r_0}\right) \qquad (7)$$

The Stability Criterion.

5. To discuss the stability of the star, we eliminate δT from equations (4) and (7), obtaining an equation in δr only. After slight simplification this is found to be

$$\begin{aligned} &C_v\frac{d^3}{dt^3}(\delta r) + \frac{(7+n-\beta)G_0}{T_0}\frac{d^2}{dt^2}(\delta r) \\ &\quad + \frac{\gamma M_r}{r_0^3}\left[\frac{\lambda+4}{\lambda+1}\left(\frac{3p}{\rho T_0} - C_v\right) + \frac{\lambda}{\lambda+1}3sC_v\right]\frac{d}{dt}(\delta r) \\ &\quad + \frac{\gamma M_r}{r_0^3}\frac{G_0}{T_0}\left[\frac{\lambda+4}{\lambda+1}(3\alpha+\beta-n) + \frac{\lambda}{\lambda+1}(7+n-\beta)3s\right](\delta r) = 0 \end{aligned} \qquad (8)$$

The various independent oscillations have time factors of the form $e^{\theta t}$ and the values of θ are obtained, in the usual way, by replacing d/dt by θ in equation (8), and treating the resultant equation as an equation for θ.

6. If each molecule, atom, or other separate unit of the gas has n degrees of freedom apart from its motion of translation, the energy per unit volume is

$$\tfrac{1}{2}(n+3)NRT + aT^4,$$

where N, R have their usual kinetic-theory meanings. By differentiation, we obtain

$$\rho C_v = \tfrac{1}{2}(n+3)NR + 4aT^3.$$

The ratio of specific heats of the stellar matter is as usual,

$$\gamma = 1 + \frac{2}{n+3}.$$

Let us allow for deviations from the gas laws by supposing the gas pressure to be $(1+\xi)NRT$, where ξ is positive, then the total pressure p is given by

$$p = (1+\xi)NRT + \tfrac{1}{3}aT^4.$$

From these equations we readily obtain

$$C_v = \frac{NR}{\rho}\left[\frac{1}{\gamma - 1} + \frac{12(1+\xi)}{\lambda}\right]$$

$$\frac{3p}{\rho T} - C_v = \frac{NR}{\rho}\left[\frac{3\gamma - 4}{\gamma - 1} + 3\xi - \frac{9(1+\xi)}{\lambda}\right]$$

Replacing NR/ρ by its value $R/m\mu$ and substituting θ for d/dt, equation (8) becomes

$$\frac{R}{m\mu}\left[\frac{1}{\gamma - 1} + \frac{12(1+\xi)}{\lambda}\right]\theta^3 + \frac{G_0}{T_0}(7 + n - \beta)\theta^2$$
$$+ \frac{\gamma M_r}{r_0^3}\left(\frac{R}{m\mu}\right)\left[\frac{\lambda + 4}{\lambda + 1}\left(\frac{3\gamma - 4}{\gamma - 1} + 3\xi - \frac{9(1+\xi)}{\lambda}\right) + \frac{3s\lambda}{\lambda + 1}\left(\frac{1}{\gamma - 1} + \frac{12(1+\xi)}{\lambda}\right)\right]\theta$$
$$+ \frac{\gamma M_r}{r_0^3}\left(\frac{G_0}{T_0}\right)\left[\frac{\lambda + 4}{\lambda + 1}(3\alpha + \beta - n) + \frac{3s\lambda}{\lambda + 1}(7 + n - \beta)\right] = 0 \quad . \quad . \quad (9)$$

7. Write this equation in the form

$$f(\theta) \equiv A\theta^3 + B\theta^2 + C\theta + D = 0 \quad . \quad . \quad (10)$$

As θ decreases from $+\infty$ to 0, $f(\theta)$ changes from $+\infty$ to D, since A is necessarily positive. If D is negative the equation must have at least one real positive root for θ and the star is unstable. Thus a first condition for stability is that D shall be positive.

As θ increases from $-\infty$ to 0, $f(\theta)$ changes from $-\infty$ to D. So that with D positive there must be at least one negative real root $\theta = -\epsilon$, and the equation assumes the form

$$A(\theta^2 + 2P\theta + Q)(\theta + \epsilon) = 0 \quad . \quad . \quad . \quad (11)$$

in which ϵ and Q are both positive. Apart from the root $\theta = -\epsilon$, the two remaining roots are

$$\theta = -P \pm (P^2 - Q)^{\frac{1}{2}}.$$

If P^2 is greater than Q, both roots are real and of the same sign as $-P$. If P^2 is less than Q, both roots are imaginary and have $-P$ for their real part. In either event, the star is stable if P is positive and is unstable if P is negative.

Comparing coefficients in the equivalent equations (10) and (11)—

$$\frac{A}{A} = \frac{B}{A(2P + \epsilon)} = \frac{C}{A(2P\epsilon + Q)} = \frac{D}{A\epsilon Q} \quad . \quad . \quad (12)$$

Since ϵ and Q are both positive, we see that if $BC<AD$, P must be negative and the star is unstable. A change from stability to instability (with D kept positive) can only occur when P vanishes, and this occurs only when $BC = AD$. Hence the necessary and sufficient condition for stability is that D and $BC - AD$ shall both be positive, or

$$BC/A > D > 0.$$

8. To assign a physical interpretation to these conditions, let us examine the critical case of P = 0. The roots for θ are now

$$\theta = \pm i\,Q^{\frac{1}{2}},\ -\epsilon,$$

where, from equations (12)

$$\epsilon = \frac{B}{A} = \frac{G_0}{C_v T_0}(7 + \nu - \beta) \quad . \quad . \quad . \quad (13)$$

$$Q = \frac{C}{A} = \frac{\gamma M_r}{r_0^3}\left[\frac{\lambda+4}{\lambda+1}\frac{(3\gamma-4)\lambda + 3\xi(\gamma-1)\lambda - 9(\gamma-1)}{\lambda+12(1+\xi)(\gamma-1)} + \frac{3s\lambda}{\lambda+1}\right] \quad (14)$$

Clearly the value of ϵ is such that the time factor $e^{-\epsilon t}$ changes at a rate comparable with that in a star contracting under gravity with no intrinsic generation of energy. Thus $1/\epsilon$ is of the order of millions of years for a dwarf star and of thousands of years for a giant; if the star for which ϵ is negative is started expanding or contracting, the motion increases exponentially, with this slow time-scale, until stable conditions are reached.

If we replace M_r by its value $\frac{4}{3}\pi\bar{\rho}r_0^3$, we see that $Q^{\frac{1}{2}}$ is of the same order of magnitude as $(\gamma\bar{\rho})^{\frac{1}{2}}$, so that the time factor $e^{\pm iQ^{\frac{1}{2}}t}$ represents vibrations of the same period as the ordinary vibrations of the star regarded as a sphere of gas, a period of days or hours. Incidentally we notice that at its extreme limit (P = 0) of stability a star can execute vibrations of this period without any decay of amplitude. These are the vibrations which I have suggested may be found to provide an explanation of the long-period variables. If P becomes negative, the amplitude of the vibrations increases exponentially with a time-scale of a few hours or days, and the star may be described as being explosively unstable.*

The Mode of Liberation of Stellar Energy.

9. Replacing A, B, C, D by their values, the condition (D<BC/A) that there shall be no explosive vibrations becomes

$$\beta + 3\alpha - n < (\tfrac{7}{3} + \alpha)\left(3 - \frac{1}{(\gamma-1)(1+\xi)} - \frac{9}{\lambda}\right)\frac{\lambda}{\lambda+1} \quad . \quad (15)$$

The condition that there shall be no unstable contraction or expansion (D>0) is

$$\beta + 3\alpha - n > -(\tfrac{7}{3} + \alpha)\frac{9s\lambda}{\lambda+4-3s\lambda} \quad . \quad . \quad (16)$$

except that, when s is so large as to make the denominator on the right negative, the inequality must be reversed.

For values of s less than this, equations (15) and (16) show that the range of values for β which are consistent with stability is restricted at both ends, the total length of range being

$$(\tfrac{7}{3} + \alpha)\left[\left(3 - \frac{1}{(\gamma-1)(1+\xi)} - \frac{9}{\lambda}\right)\frac{\lambda}{\lambda+1} + \frac{9s\lambda}{\lambda+4-3s\lambda}\right] \quad (17)$$

* *M.N.*, **85** (1925), p. 929.

If this expression were to vanish or become negative for any particular star, the two ranges of instability would have met and crushed all stable configurations out of existence. Hence expression (17) must be positive for every star which continues to exist stably in the sky.

For ordinary stellar matter γ is about $1\frac{2}{3}$, so that the expression in square brackets will be positive for stars of small mass, for which λ is large. Thus $\frac{7}{3} + \alpha$ must be positive. But as we pass to stars of larger mass λ decreases, and at about $\lambda = 6$ (mass about 2 ⊙) we find that the expression in square brackets becomes negative unless we assign positive values to s and ξ. In other words, moderately massive stars cannot rest in stable equilibrium until there are substantial deviations from the gas laws.

10. Somewhere in the range between $\lambda = 6$ and $\lambda = 3$, according to the value of ξ, the bracket on the right of inequality (15) becomes negative. Thus for stars of the corresponding mass, it appears that there will be explosive instability unless $\beta + 3\alpha < n$.

This rules out large values of $\beta + 3\alpha$, at least for stars of large mass, and so presumably for stars of all masses, since the values of β and α can hardly depend on the mass of the star. Thus a star in which the rate of generation of energy increased rapidly with the temperature and density would be explosively unstable.

For instance, if energy were generated as a consequence of collisions, the frequency of collisions per unit mass would be proportional to $\rho T^{\frac{1}{2}}$, and this factor alone would give values $\beta = \frac{1}{2}$, $\alpha = 1$. The value of β would almost certainly need a substantial increase to allow for the extra violence of collisions at high temperatures, but even without this, $\beta + 3\alpha$ would have a value of 3·5, which is too large for the inequality $\beta + 3\alpha < n$ to be satisfied.

From inequality (15) we have found that $\beta + 3\alpha$ cannot have a large positive value while equation (16) shows that it cannot have a large negative value. We conclude that $\beta + 3\alpha$ must be numerically small.

11. When once the possibility of energy being generated by collisions can be ruled out, there are strong physical reasons for expecting the generation of energy to be a spontaneous process proceeding at a rate independent of both temperature and density—*i.e.* that $\alpha = \beta = 0$.

This is precisely the conclusion to which our stability investigation leads. The value of $3\alpha + \beta$ has been found to be numerically small and this requires that the rate of generation of energy shall be approximately proportional to $(T^3/\rho)^c$, where c is some constant. It seems impossible to reconcile any value of c other than $c = 0$ with either the general principles of physics or with the evidence of observational astronomy.

Consequently from here onwards we put $\alpha = \beta = 0$, thereby assuming that the rate of liberation of stellar energy is approximately independent of both temperature and density. In brief, we assume our star to be built on the "uranium" model.

The Stability Criterion for a "Uranium" Star.

12. On putting $\alpha = \beta = 0$, equation (15) gives as the condition that there shall be no explosive instability—

$$n + \tfrac{7}{3}\left(3 - \frac{1}{(\gamma-1)(1+\xi)} - \frac{9}{\lambda}\right)\frac{\lambda}{\lambda+1} > 0 \quad . \quad . \quad (18)$$

For stars of moderate mass this is satisfied automatically, although obviously it may fail for very massive stars. It is conceivable that these can only rest stably in exceptional configurations in which the effective value of n is unusually large. Or, again, it is possible that viscosity may check the onset of instability, the stars pulsating as long-period variables.

Equation (9) gives as the condition that there shall be no contractional instability (D>0),

$$\frac{3s\lambda}{\lambda+4}(7+n) > n \quad . \quad . \quad . \quad . \quad (19)$$

which of course requires a positive finite value for s. With $n = \frac{1}{2}$, s must be greater than 0·155 for a star in which $\lambda = \frac{2}{3}$ (mass about 20 ⊙) and greater than 0·027 for a star in which $\lambda = 20$ (mass about ⊙). A star which at any instant finds itself in a configuration in which s is less than the value given by this equation, cannot rest in stable equilibrium; it will contract or expand until it has reached such a density that inequality (19) is satisfied.

The minimum values we have just obtained for s are numerically small, but this does not necessarily mean that the star behaves nearly like a perfect gas. The pressure is produced by atoms and electrons, and in a perfect gas each atom and each electron contributes equally to the pressure. Now the electrons are some 50 or 100 times as numerous as the atom, and their large contribution to the pressure will always obey the gas laws to a high degree of approximation. Consequently all deviations from the gas laws must arise from the pressure of the comparatively few atoms. And for these substantially to disturb the gas laws, their own pressure must be very high. It is probably legitimate to suppose that when stability is reached these are packed together so as to be almost touching, just as ordinary atoms or molecules are in a solid or a liquid. Equation (19) then means that a star will go on contracting, as in the Helmholtz-Kelvin contraction theory, until its central regions have become almost liquid or solid in character, providing a comparatively unyielding base on which the upper gaseous layers of the star may rest.

Equation (19) has only been proved for the special model star under consideration. But in my earlier paper I showed accurately and quite generally that contractional stability or instability depended solely on the sign of

$$\frac{\partial(\mathrm{E}-\mathrm{G})}{\partial \mathrm{T}} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where E, G are the rates of emission and generation for any star whatever. For the special model under consideration the sign of expression (19) is

the same as that of − D which, as we have seen, must be negative for stability. Hence the general condition that any star shall be contractionally stable is that expression (20) shall be *negative* and a "uranium" star is contractionally stable only if $\partial E/\partial T$ is *negative*.*

Areas of Stability.

13. All possible configurations for a star can be represented in a Russell diagram; of these configurations some will be stable and some unstable. When actual stars are plotted on such a diagram, some areas, representing unstable configurations, will be devoid of stars. The stars will cluster in certain areas of concentration, which we may designate "areas of stability."

Stars inside these areas of stability are in conditions such that there are substantial deviations from the gas laws. This makes it impossible to discuss conditions inside these areas, except with more refined analysis than we have at present available. But no such difficulty arises outside these areas, and we can investigate the boundaries of the "areas of stability" by regarding them as the curves in the diagram along which the ordinary gas laws first begin to fail. For this investigation it is of course legitimate to assume the ordinary gas laws.

14. According to a formula of R. H. Fowler,† the proportion x of atoms which are completely ionised down to the τ-quantum ring of electrons in atoms of atomic number N is given by ‡

$$\log\left(\frac{x}{1-x}\rho\right) = -\frac{69000N^2}{T(\tau+1)^2} + \tfrac{3}{2}\log T + \log \tfrac{1}{2}\mu + \log\tfrac{1}{2}\sigma - 7{\cdot}790 \quad (21)$$

where all logarithms are to the base 10.

If λ denote the ratio of gas pressure to pressure of radiation,

$$\frac{R}{m\mu}\rho T = \lambda(\tfrac{1}{3}aT^4), \quad . \quad . \quad . \quad . \quad (22)$$

whence

$$\log\rho = 3\log T + \log(\lambda\mu) - 22{\cdot}511 \quad . \quad . \quad (23)$$

From these we obtain at once

$$\log\left(\frac{x}{1-x}\right) = -\frac{69000N^2}{T(\tau+1)^2} - \tfrac{3}{2}\log T + \log\frac{\sigma}{4\lambda} + 14{\cdot}721 \quad (24)$$

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* My earlier paper gave the wrong signs to these criteria as a consequence of determinating them (*M.N.*, **85** (1925), p. 923) from a special case which, as I now see, was not one of contractional but of explosive instability (not e^{kt} with k large, but $e^{kt}\cos qt$). If dG/dT is large and positive the star is, as we have seen, unstable, but it does not follow that a star is in general unstable if expression (20) is negative. Russell and Eddington have always maintained that, in the special case of the gas laws being perfectly obeyed, a star would be unstable if expression (20) were positive. I gladly concede that in this special case their statement was correct and my own contrary statement incorrect. But I still do not find their argument convincing, and it fails to show that there is instability when dG/dT is large and positive; indeed Russell's 1925 theory of stellar evolution was based on the supposition that there is stability in this case.

† *M.N.*, **83** (1923), p. 407.

‡ *M.N.*, **85** (1925), p. 928.

Denoting the right hand by ϕ, we have

$$1 - x = (1 + 10^{\phi})^{-1} \quad . \quad . \quad . \quad (25)$$

so that

$$\log[(1 - x)\rho] = \log \rho - \log (1 + 10^{\phi}) \quad . \quad . \quad (26)$$

To simplify the problem, suppose that all the atoms have the same atomic number N ; then $1 - x$ is the proportion of atoms which retain some at least of the electrons of their $(\tau + 1)$-quantum ring, and $(1 - x)\rho$ is proportional to the total space per unit volume occupied by these atoms. We may reasonably disregard, by comparison, the space occupied by more highly ionised atoms. Thus $(1 - x)\rho$ gives a measure of the total space occupied by all the atoms in unit volume, being, roughly speaking, proportional to the b of Van der Waals' equation. The boundary of an area of stability will be a curve along which $(1 - x)\rho$ has a finite and approximately constant value.

As a star of given mass changes its radius and temperature, λ and μ will change less rapidly than other quantities and may, to a first approximation be treated as constants. Equation (24) now shows that x is zero both at zero and at infinite temperature ; at the latter temperature, the density is so great, when the gas laws are assumed to be obeyed, that no atoms are ionised, although naturally in an actual star the gas laws would fail before such conditions were reached. Equation (26) shows that $(1 - x)\ \rho$ increases from 0 to $+\infty$ while T increases from 0 to ∞, although again the conditions at $T = \infty$ could not prevail in an actual star.

As the value of $(1 - x)\ \rho$ increases from 0 to ∞, it may pass through maxima or minima. Differentiation of equation (26), treating λ and μ as constants, gives

$$\frac{\partial}{\partial T} \log[(1-x)\rho] = \frac{3\times 0{\cdot}4343}{T} - \frac{10^{\phi}}{1 + 10^{\phi}}\left[\frac{69000N^2}{T^2(\tau + 1)^2} - \frac{3\times 0{\cdot}4343}{2T}\right] \quad (27)$$

If $(1 - x)\rho$ passes through maxima or minima, these will occur when the right-hand member of this equation vanishes, and this is when

$$\frac{35000N^2}{(\tau + 1)^2 T} = 1 + \tfrac{2}{3}10^{-\phi} \quad . \quad . \quad . \quad (28)$$

15. In an area in which the gas laws are obeyed, x will, in general, be nearly equal to unity, so that equation (25) shows that 10^{ϕ} must be large. If we momentarily neglect $10^{-\phi}$, equation (28) becomes

$$T = \frac{35000N^2}{(\tau + 1)^2} \quad . \quad . \quad . \quad . \quad (29)$$

and it is readily found that $(1 - x)\rho$ passes through a minimum when T has this value. Thus as T increases from 0 to ∞, the gas laws are most nearly obeyed (or, more precisely, the b of Van der Waals' equation is smallest) when T has this value.

As a star contracts and its temperature increases, the various rings of electrons are set free in turn. At one instant the L-rings will be

left un-ionised, their combined volume causing a certain degree of congestion in the star's interior.

Further contraction with increasing temperature strips more and more atoms free of their L-electrons, thus relieving the congestion, until a minimum congestion is reached when the temperature attains the value defined by equation (29) with $\tau + 1$ put equal to 2, since the L-electrons describe 2-quantum orbits. Passing through this minimum the congestion goes on increasing—to infinity according to our equations, but in actual fact, of course, until the gas laws are no longer obeyed and our equations break down.

When we can next trace the progress of events, the K-ring electrons are being set free, and the congestion reaches a minimum when τ is given by equation (29) with $\tau + 1 = 1$. After this minimum the congestion continually increases until the star ends with practically all its atoms stripped bare down to their nuclei.

Thus the ordinary sequence of events for a star is a succession of alternations of high congestion and the reverse, the various minima of congestion being given by equation (29) with $\tau + 1$ put equal to 1, 2, 3 . . . These temperatures are independent of the luminosity of the star, so that in the Russell diagram we ought to find alternating strips of high congestion (stability, occupied by stars) and of low congestion (instability and absence of stars), these strips lying parallel to the lines of constant central temperature. The areas of stability ought thus to be strips or elongated patches having lines of constant central temperature for their lines of maximum concentration.

It is easy to pick out just such patches. The principal is, of course, the " main-sequence " along which Russell has calculated that the central temperature is fairly constant. His calculations * gave an almost uniform central temperature of about 30,000,000°, although my own calculations,† which assumed a less special model for the star, indicated a less uniform central temperature averaging about 60,000,000° for stars of moderate mass. On the cool side of the main sequence is another patch of stars (the " giant " branch) representing a central temperature of about 20,000,000°, while on the hot side of the main sequence lie the white dwarfs of unknown central temperature.

These areas of stability are readily identified with those demanded by our theory. The main sequence must clearly represent K-ring congestion, the area on the cool side (the giant branch) representing L-ring congestion, while the area of the white dwarfs represents a congestion of the nuclei and of those few K-rings which have escaped ionisation. If we suppose the main sequence to have a central temperature of about 60,000,000° and the giant branch to have a central temperature of about 20,000,000°, we may suppose the temperature of minimum L-ring congestion to be about 40,000,000°. Giving T this value and putting $\tau + 1 = 2$, equation (29) gives N = 68 as the mean atomic number of the atoms concerned. On the other hand, Russell's calculations, referring to a star whose generation of energy is concentrated to a substantial degree near its centre (Eddington's model

* *Nature*, 1925 August 8.

† *M.N.*, **87** (1926), p. 40.

$k\eta$ = cons.) would assign temperatures of about 30,000,000° and 6,000,000° to the two branches. The temperature of minimum L-ring congestion would now be of the order of only 15,000,000°, and the resulting value of N would only be about 42.

When the term $\frac{8}{3}10^{-\phi}$ is restored in equation (28), the calculated value of T is necessarily diminished. Numerical calculation shows that this term is unimportant except for K-ring congestion, where it becomes important for stars of small mass. Indeed it is so important, that for stars of mass below a certain limit equation (28) has no root at all; the minimum of congestion disappears entirely and the stable areas of L-ring and K-ring congestion run into one another, giving the characteristic "inverted γ" appearance to the Russell diagram.

The general scheme suggested by our theory is shown in fig. 1, which is drawn on the background of Seares' diagram for the absolute magnitudes and spectral types of 2000 stars.

Stellar Evolution.

16. Although the detailed problems of stellar evolution lie outside the scope of the present paper, we may glance briefly at the general course of evolution indicated by our theory.

A star, when first formed out of a nebula, has a density so low that the gas laws are accurately obeyed, and is consequently unstable. Stability is acquired only after contraction to such a density that there are substantial deviations from the gas laws. Assuming the star to have the mass of a giant star, the whole period of this contraction from the nebular to the giant state is only a few thousand years, and, astronomically speaking, the star may be expected to flash out as a red giant almost immediately after its birth.

The star is now in one of the areas of stability in our diagram. Its mass will decrease owing to loss by radiation; its radiative power per unit mass will decrease owing to those atoms which produce most radiation using themselves up first, while the inert atoms are hardly annihilated, if at all. These two factors cause the star to move down and through the area of stability until it again becomes unstable through emerging from the area on one side or the other. If it emerges on the hotter side it contracts until it reaches the next hotter area of stability, and this process may be continually repeated until finally it may drop off the main sequence into the white-dwarf area of stability. Consequently the white-dwarf series ought to extend upwards to luminosities comparable with those of the brightest of stars. The Wolf-Rayet stars seem capable of falling into position in the top half (high luminosity) of the white-dwarf area of stability.

I have investigated the general path in a Russell diagram of a star which is losing mass and radiating power simultaneously, and found * that at first the star would contract in the way just described; our present view leads us to regard the contraction as consisting mainly of a series of drops rather than of a continuous evolution, but the main trend of the path must be much the same. But it was shown that after

* *M.N.*, **85** (1925), p. 401.

a time the star might again begin to expand, and this possibility is also present in the theory just outlined. When a star is contracting through a region of instability it is emitting more radiation than it generates, the deficit being made good by drawing upon the star's

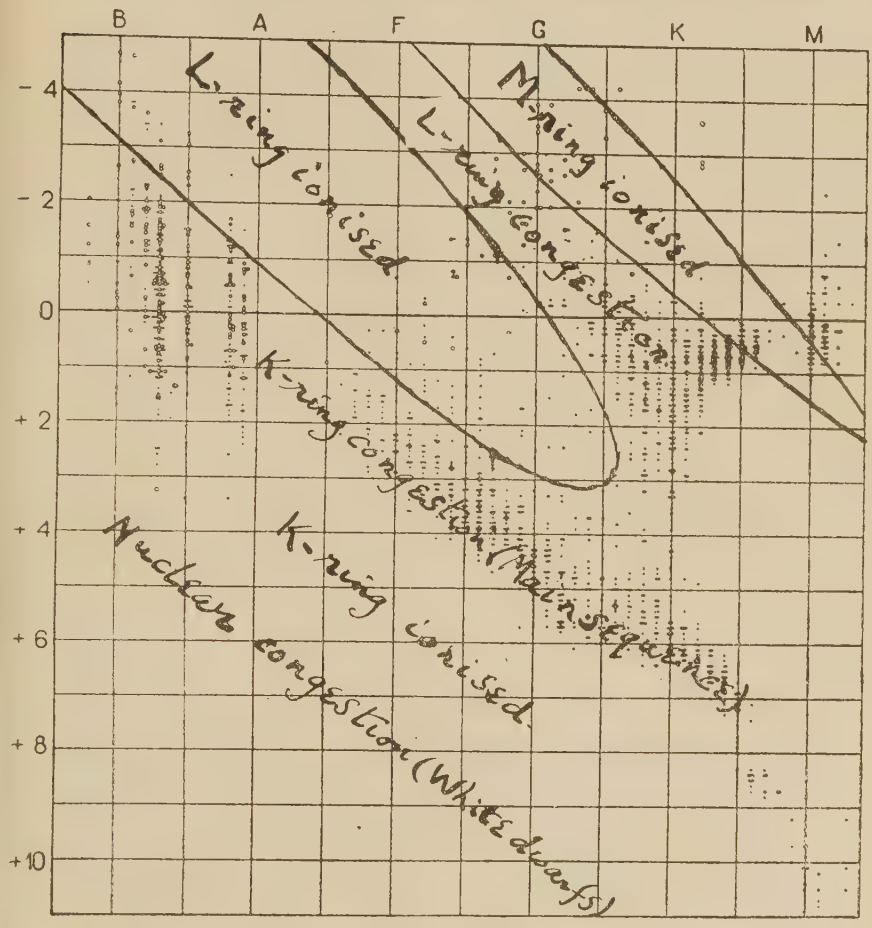


FIG. 1.

gravitational potential energy. In the same way when the star is expanding it is generating more energy than it emits, the surplus going to increase its gravitational potential energy.

I further found * that the process of fission must throw the two components to a region of the diagram on the hot side of the region occupied by the parent star. On our present theory this would involve

* *M.N.*, **85** (1925), p. 797.

a drop to a new region of stability on the hot side of the region originally occupied. This would explain the preponderance of binaries on the upper half of the main sequence (B-type stars), and would demand a corresponding preponderance on the upper half of the white-dwarf sequence (O-type stars).

Actual stars must consist of mixtures of elements of differing atomic weights, and this must result in a blurring of the sharp outlines of the map of stable and unstable configurations; a crucial test of the theory will be its ability to survive this blurring without losing its capacity to explain the observed distribution of the stars. Moreover, although it is easy in theory to obtain a sharply-defined area of instability between the main sequence and the white-dwarf sequence, it is not easy to do so with numerical values that seem to be at all plausible. Probably a proper discussion of this problem demands a more accurate treatment of deviation from the gas laws, while it is also possible that there may be some physical property of bare nuclei which has not been taken into account in our present investigation. The theory which I have outlined seems to offer such promise of solving one of the most puzzling problems of astronomy that I have thought it legitimate to put it forward without delaying to discuss various possible difficulties.

The Atmosphere of a Pulsating Star. By Dr. J. J. M. Reesinck.

(Communicated by the Secretaries.)

1. In his articles on the pulsation theory of the Cepheid variables (*M.N.*, **79**, 2, 1918; *M.N.*, **79**, 177, 1919) Professor A. S. Eddington has tried to explain why the maxima of temperature and of radiation occur, not at the moment of minimum volume, but at that of maximum velocity of expansion. In the interior of the star, where the pulsation goes on adiabatically, the maximum temperature would coincide exactly with the minimum volume. Since, however, according to Professor Eddington the principal cause of the temperature variations of the outer layers of the star is the radiation from the interior, these variations would be somewhat retarded in these outer layers. Professor Eddington's opinion was that the heat capacity of these layers would be sufficient to explain a delay of phase of the amount observed.

Dr. J. H. Jeans (*M.N.*, **86**, 574, 1926), however, pointed out that the heat capacity of the surface layers of a star is very small in comparison with the total radiation in one period. So it is rather improbable that a considerable retardation of the temperature variation occurs. But Jeans considers only the very thin layer in which the influence of the boundary condition for the radiation is not entirely negligible; it is possible that the non-adiabatic region meant by Eddington extends to a much greater depth.*

* The objections I made in *M.N.*, **86**, 379, against Jeans' argument, however, did not concern his neglecting the energy of the surface disturbance but his neglecting λT_2 in comparison with T_1, which, as I showed, is *petitio principii*.

In this paper the atmosphere of a pulsating star is treated theoretically, and it will be shown that differences in phase of importance do not occur.

2. The following simplifying suppositions are introduced :

(1) We neglect the curvature of the atmospheric layers. The acceleration of gravity g is the same everywhere in the atmosphere. During the pulsation, however, g is variable, the atmosphere is thinned out over a larger surface, and it has, as a whole, an acceleration $\ddot{R}$ where R is the radius of the star.

(2) With Schwarzschild we suppose that there is only radiation in two directions, a radial outward flow of heat I_1 and a radial inward flow of heat I_2.

(3) The pressure of radiation is neglected.

(4) In the state of equilibrium there is no production of energy in the layers considered.

(5) The coefficient of absorption k is supposed to be a constant always and everywhere in the atmosphere.

(6) The variation of internal energy (*e.g.* the energy of ionisation) is neglected.

(7) Only infinitesimal, simply harmonic pulsations are considered.

3. Let T be the temperature, p the pressure, ρ the density, h the height reckoned from an arbitrary zeroplane, t the time, the zero point of which will be chosen afterwards. We denote by c_p and c_v the specific heat at constant pressure and at constant volume respectively, and by γ their ratio. By μ we denote Stephan's constant and by H the gas constant for a unit of mass.

The equations for the radiation are :

$$\frac{dI_1}{dh} = -I_1 k\rho + \tfrac{1}{2}\mu T^4 k\rho + \tfrac{1}{2}\epsilon\rho \quad . \quad . \quad . \quad (1)$$

$$\frac{dI_2}{dh} = +I_2 k\rho - \tfrac{1}{2}\mu T^4 k\rho - \tfrac{1}{2}\epsilon\rho \quad . \quad . \quad . \quad (2)$$

where ϵ is that part of the energy produced by the pulsation, which is not used in raising the temperature (reckoned per unit of mass and time). ϵ is given by the equation :

$$-\epsilon\rho dh = \left(c_v\frac{dT}{dt} + p\frac{d}{dt}\frac{1}{\rho}\right)\rho dh$$

or

$$\epsilon = -c_v\dot{T} + \frac{HT}{\rho}\dot{\rho} \quad . \quad . \quad . \quad . \quad (3)$$

The total amount of radiation is $I_1 + I_2 = \mu T^4$. Addition and subtraction of (1) and (2) gives for the total amount of radiation and for the resulting outward flow of energy $S = I_1 - I_2$, the equations :

$$\frac{dS}{dh} = -c_v\dot{T}\rho + HT\dot{\rho} \quad . \quad . \quad . \quad (4)$$

$$\frac{d}{dh}(\mu T^4) = -k\rho S \quad . \quad . \quad . \quad . \quad (5)$$

Let $x - 1$ be the optical depth,* defined in the usual way, then $dx = -k\rho dh$. Substituting this in equations (4) and (5) and eliminating S from these equations we get :

$$\mu k\frac{d^2}{dx^2}T^4 = c_v\dot{T} - HT\frac{\dot{\rho}}{\rho} \quad . \quad . \quad . \quad (6)$$

As a consequence of our supposition under (7) above we may put :

$$T = T_0(1 + T_1) = T_0(1 + T_s \sin nt + T_c \cos nt),$$
$$p = p_0(1 + p_1) = p_0(1 + p_s \sin nt + p_c \cos nt), \text{ etc.} \quad . \quad (7)$$

The frequency n depends on the conditions in the interior of the star and must therefore in the present problem be considered as a given quantity. The zero point of time may be chosen so that in the interior of the star the cosine terms vanish. Then we have, *e.g.* $R_c = g_c = 0$.

Since k is assumed constant we may write :

$$R^2dx = R_0^2dx_0,$$

therefore :

$$dx_0 = (1 + 2R_1)dx \quad . \quad . \quad . \quad . \quad (8)$$

Substituting the expressions (7) and (8) in equation (6) we obtain :

$$\mu k\frac{d^2}{dx_0^2}T_0^4 + 4\mu kR_1\frac{d^2}{dx_0^2}T_0^4 + 4\mu k\frac{d^2}{dx_0^2}(T_0^4T_1) - c_vT_0\dot{T}_1 - HT_0\dot{\rho}_1 \quad (9)$$

The term independent of the time, when put equal to zero, gives Schwarzschild's equations :

$$\frac{d^2}{dx_0^2}T_0^4 = 0 \quad . \quad . \quad . \quad . \quad (10)$$

and

$$T_0^4 = T_{b0}^4x_0 \quad . \quad . \quad . \quad . \quad (11)$$

where T_{b0} is the boundary temperature of the star in the state of equilibrium. The remaining terms of (9) give, with application of (10) and (11) :

$$4\mu kT_{b0}^4\frac{d^2}{dx_0^2}(T_1x_0) = c_vT_0\dot{T}_1 - HT_0\dot{\rho}_1 \quad . \quad . \quad (12)$$

Since $p = H\rho T$ we have $\rho_1 = p_1 - T_1$ and therefore $\dot{\rho}_1 = \dot{p}_1 - \dot{T}_1$. Substituting this expression for $\dot{\rho}_1$ in (12), putting $H = c_p - c_v$ and taking the sine and cosine terms separately, we get :

$$\frac{d^2}{dx_0^2}(T_sx_0) = -ax_0^{1/4}\left(T_c - \frac{\gamma - 1}{\gamma}p_c\right) \quad . \quad . \quad (13)$$

$$\frac{d^2}{dx_0^2}(T_cx_0) = +ax_0^{1/4}\left(T_s - \frac{\gamma - 1}{\gamma}p_s\right) \quad . \quad . \quad (14)$$

where $a = c_vn/4\mu kT_{b0}^3$.

* The meaning of the letter x is therefore not the same as in Schwarzschild's articles.

4. The dynamical condition is :

$$\frac{dp}{dh} = -g\rho - \rho\ddot{\mathrm{R}}.$$

Since by (8) $\rho_0 dh_0 = \rho dh(1 + 2\mathrm{R}_1)$ this equation reduces to

$$(1 + 2\mathrm{R}_1)\frac{dp}{\rho_0 dh_0} = -g - \ddot{\mathrm{R}}.$$

The terms not containing the time give :

$$dp_0 = g_0 - \rho_0 dh_0.$$

With application of this equation the remaining terms reduce to

$$-g_0\frac{d(p_0p_1)}{dp_0} = -g_1g_0 + n^2\mathrm{R}_0\mathrm{R}_1 + 2g_0\mathrm{R}_1,$$

or, since $g_1 = -2\mathrm{R}_1$:

$$d(p_0p_1) = -\mathrm{R}_1\Big(4 + \frac{n^2\mathrm{R}_0}{g_0}\Big)dp_0.$$

At the boundary $p_0 = p_0p_1 = 0$, thus by integrating we obtain :

$$p_1 = -\mathrm{R}_1\Big(4 + \frac{n^2\mathrm{R}_0}{g_0}\Big) \quad . \quad . \quad . \quad . \quad (15)$$

which is the boundary condition given by Eddington.

Separating the sine and cosine terms we get :

$$p_s = -\mathrm{R}_s\Big(4 + \frac{n^2\mathrm{R}_0}{g_0}\Big); \qquad p_c = 0 \quad . \quad (16), (17)$$

5. The term $(\gamma - 1)p_s/\gamma$ in (14) admits of a simple interpretation. It is the constant value which T_s would have in the case of an adiabatic pulsation of the atmosphere. Therefore we put, in agreement with (16) :

$$\mathrm{T}_s(ad) = \frac{\gamma - 1}{\gamma}p_s = -\frac{\gamma - 1}{\gamma}\Big(4 + \frac{n^2\mathrm{R}_0}{g_0}\Big)\mathrm{R}_s.$$

By (17) $p_c = 0$. Thus if we write :

$$\{\mathrm{T}_s - \mathrm{T}_s(ad)\}x_0 = y; \qquad \mathrm{T}_c x_0 = z$$

the equations (13) and (14) reduce to :

$$\frac{d^2y}{dx_0^2} = -azx_0^{-3/4}; \qquad \frac{d^2z}{dx_0^2} = +ayx_0^{-3/4} \quad . \quad (18), (19)$$

6. The four constants of integration of the differential equations (18), (19) are fixed by two conditions for large values of x and two conditions at the boundary. For large x

$$\mathrm{T}_s = \mathrm{T}_s(ad); \qquad \mathrm{T}_c = 0 \quad . \quad . \quad (20), (21)$$

At the boundary there is no inward flow of energy, therefore the total radiation μT^4 must be equal to the resulting outward stream S. So by (5) the boundary condition for $x = 1$ becomes

$$\mu T^4 = \frac{d}{dx}(\mu T^4).$$

Applying (8) and (11) and omitting terms not containing a time factor we obtain after some reduction :

$$R_1 + 2\frac{dT_1}{dx_0} = 0$$

or, separating the sine and cosine terms :

$$\frac{dT_s}{dx_0} = -\tfrac{1}{2}R_s\,; \qquad \frac{dT_c}{dx_0} = 0 \quad . \quad . \quad (22), (23)$$

The second member of (22) prevents the temperature gradient from changing by the variation in thickness of the outer layers, so as to violate the boundary condition for the radiation.

7. We now make an estimate of the order of magnitude of the constant a occurring in the equations (13) and (14). With a molecular weight 2 and $\gamma = 5/3$ we get $c_p = 2H/3 = 2{\cdot}8\times10^7$ cm.^{2} sec.$^{-2}$; $\mu = 10{\cdot}6\times10^{-5}$ gr. sec.$^{-3}$. Take $T_{b0} = 6000°$ and the period $P = 5{\cdot}4$ days $= 4{\cdot}5\times10^5$ sec. ; if for k we take 1 gr. cm.$^{-2}$, which is certainly much below the real value, we find that a is a pure number of the order of 10^{-5}.

Since a may be written in the form :

$$a = \frac{\pi c_p T_{b0}}{2k\mu T_{b0}{}^4 P}$$

it is evident that this constant is of the order of the ratio of the heat capacity of an element at the boundary of the star and the radiation of that element during a period. Since a is small, every element must absorb and re-emit in a period an amount of energy which is large in comparison with its own store of energy. So we may expect already that the delay of phase supposed by Eddington will not be very important.

8. In order to integrate the equations (18) and (19) put :

$$y + iz = u\,; \quad y - iz = v.$$

After multiplication of equation (19) by i, addition and subtraction give the equations :

$$\frac{d^2u}{dx_0{}^2} = +iaux_0{}^{-3/4}\,; \qquad \frac{d^2v}{dx_0{}^2} = -iavx_0{}^{-3/4},$$

the general solution of which is :

$$u = x_0{}^{1/2}\{AJ_{+4/5}(\tfrac{8}{5}\sqrt{-ai}x_0{}^{5/8}) + BJ_{-4/5}(\tfrac{8}{5}\sqrt{-ai}x_0{}^{5/8})\} \quad . \quad . \quad (24)$$

$$v = x_0{}^{1/2}\{A'J_{+4/5}(\tfrac{8}{5}\sqrt{+ai}x_0{}^{5/8}) + B'J_{-4/5}(\tfrac{8}{5}\sqrt{+ai}x_0{}^{5/8})\}. \quad . \quad (25)$$

where A, B, A′, and B′ are (generally complex) constants of integration. In the convergent expansion of the Bessel functions :

$$J_n(\xi) = \text{constant} \times \xi^n \left\{ 1 - \frac{\xi^2}{4(n+1)} + \frac{\xi^4}{32(n+1)(n+2)} - \ldots \right\}$$

every term contains one factor a more than the preceding one. Therefore for $x_0 \ll 10^4$ the first term of the expansion suffices.

In that case the general form of y and z is a linear expression in x_0. It appears that the right-hand members of (18) and (19) which are due to our making allowance for the influence of radiation have no influence on the solution. Therefore leakage of heat is not sufficient to explain a difference in phase between T and R.

The general solution * of (13) and (14) is :

$$T_s = p + \frac{q}{x_0}; \qquad T_c = r + \frac{s}{x_0} \quad . \qquad . \quad (26), (27)$$

According to the boundary conditions (20), (21), (22), and (23) the constants of integration must be

$$p = T_s(ad); \qquad q = +\tfrac{1}{2}R_s; \qquad r = 0; \qquad s = 0.$$

Thus :

In the outer layers of a pulsating star there is a deviation from an adiabatic process, but there is no difference of phase between T, p, ρ, *and* R.

9. One might ask if Eddington's non-adiabatic region could extend much deeper than an optical depth of 10^{-4}. It may therefore be of some interest to show the character of the functions T_s and T_c according to (24) and (25) for large values of x. For large values of the argument the Bessel functions may be expanded in a semiconvergent series. It appears in this way that T_s and T_c become periodic functions of $x_0^{5/8}$ with a period of about 6000 and multiplied by a factor which increases exponentially with $x_0^{5/8}$.

Therefore in the outer layers there is not only nothing which *causes* a phase difference, but it is also impossible that a small deviation from an adiabatic process at great optical depth increases in the outer layers and so gives the required difference at the surface.

10. Since our conclusions are entirely based on the *smallness* of the right-hand members of (13) and (14) we may expect that the special form of those members, and therefore the special assumptions underlying this investigation, especially the assumptions mentioned in § 2, will have little influence on the result. Moreover, a rather easy calculation shows that taking into account pressure of radiation, variability of k, production of new energy or variation of ionisation energy, does not change our results in any way.

11. Eddington's assumption that in the outer layers " the temperature oscillations are determined by the heat received and radiated rather than by compression and rarefaction " is wrong, as is evident

* In *M.N.*, **86**, 379, equations of the same form were derived from Jeans' equations after taking into account terms erroneously neglected by him.

from (16).* Yet, even if we introduce this assumption, our conclusions remain unchanged. We have to put $\rho = 0$ and therefore $T_s(ad) = 0$. Instead of (13) and (14) we get :

$$\frac{d^2}{dx_0^2}(T_s x_0) = -ax_0^{1/4}T_c ; \qquad \frac{d^2}{dx_0^2}(T_c x_0) = +ax_0^{1/4}T_s.$$

The general solution is given again by (26) and (27) in the outer layers, and for large values of x_0 its general character is the same as that of the correct equations (24) and (25).

12. In Eddington's explanation of the phase difference between T and R, the enlargement of the radiating surface and the thinning out of the atmosphere at the moment of maximum expansion seem to play an important part, while in our equations (13) and (14) the quantity R_1 does not appear. The reason is that in (9) R_1 is multiplied by $d^2T_0^4/dx_0^2$, *i.e.* with the production of energy in the state of equilibrium. Since we have neglected this quantity, R_1 does not occur in the first order terms. Indeed, the effect meant by Eddington, the influence of the variation of the transparency (which variation is of the first order) on the production of energy by the pulsation (also of the first order) is an effect of the second order.

13. Conclusion :

The pulsation theory of the Cepheid variables in the form given by Eddington cannot explain the coincidence of the maximum radiation (or temperature) with the maximum velocity of approach.

The present investigation is a part of my work at the Astronomical Institute of the University of Amsterdam. I am highly indebted to the Director, Professor A. Pannekoek, for his valuable suggestions.

On the Nature of the Spiral Nebulæ. By Bertil Lindblad, Ph.D.

In his *Leçons sur les Hypothèses Cosmogoniques*, p. 262, Poincaré attempts to draw a parallel between the figure of a rotating stellar system and the figures of equilibrium of a rotating incompressible liquid. The stellar system is considered in the picture of a gaseous mass with stars as molecules. " Au début, la rotation étant nulle ou très faible, la figure de la mass gazeuse est sphérique ; la rotation s'accélérant, elle s'aplatit et prend une forme analogue a l'ellipsoïde de MacLaurin ; la rotation devenant encore plus rapide, l'aplatissement augmente, et la figure devient analogue à l'ellipsoïde de Jacobi. Si la rotation s'accroît encore, la force centrifuge aux deux extrémités du grand axe viendra à l'emporter sur l'attraction, et la matière s'échappera en deux jets à ces sommets : ces deux jets prendront évidemment une forme spirale, en vertu du principe des aires, l'*aile marchante* prenant

* *Cf.* also Eddington's remark, *M.N.*, 79, 182, footnote.

un retard sur le *pivot*. Ainsi, la masse gazeuse offrira l'image d'une nébuleuse spirale, ses molécules représentant les étoiles dont se compose cette nébuleuse."

Though these statements by the famous mathematician were only meant as a mere indication of a possible analogy, we may be allowed to take them as point of departure when trying to get a more definite view of the matter in question. The great question concerning the nature of the nebulæ seems to be at present if the "amorphous nebulosity" is really of a gaseous nature, or is only apparently irresolvable while consisting in reality of stars of somewhat smaller intrinsic brightness than the whitish stars in the outer spiral arms. This question may be considered identical with the question if the elliptic nebulæ without arms are gaseous or consist of stars. The spectra of the nuclear regions of spiral nebulæ and of elliptic nebulæ must be considered favourable for the "stellar" theory, but cannot be considered to give conclusive evidence. The final answer can only be given by direct observation, but it is my purpose here to develop some points of the "stellar" theory, partly by summarising a few results given in earlier papers.*

It is certainly straining the analogy with the liquid too far to assume that for a certain flattening of the figure of equilibrium the series of figures passes over from a form analogous to MacLaurin's ellipsoid to one analogous to Jacobi's ellipsoid. From Jeans' † theorem on the general frequency function of density and velocity in a steady state we may conclude that the rotational symmetry ought to exist as long as a steady state is possible. Further, there appears to be no room in this case for considerations of "secular stability." The analogous series of figures for the liquid ought, therefore, to be the MacLaurin series until the limit of the "ordinary stability" at eccentricity 0·9529.

The MacLaurin ellipsoids of greater eccentricity than 0·9529 are unstable for a type of two-dimensional sectorial harmonic waves ‡ in which two opposite elevations of the surface travel with a relative speed $\frac{1}{2}\omega$ around the equator of the spheroid, ω being the speed of rotation of the liquid. Waves of this type will exist also in a stellar system of uniform mean speed of rotation and of uniform density in the "galactic" regions. It is probably more than a mere chance that the observed limit of flattening of regular elliptic nebulæ at the point where these begin to merge into the spiral types seems to agree closely with the limiting eccentricity for the stability of MacLaurin's ellipsoid. Hubble § gives as upper limit of the "ellipticity" for the elliptic nebulæ 0·75, which corresponds to the eccentricity 0·968. This result is confirmed in a statistical investigation by Lundmark on the material in Reinmuth's recent Catalogue, the results of which have kindly been put at my disposal. The regular elliptic types

* *Arkiv för Matematik, Astronomi och Fysik*, Stockholm, 19 A, Nos. 21, 27, 35; 19 B, No. 7 (1925 and 1926). *Nova Acta Regiæ Soc. Sc. Upsaliensis*, Volumen extra ordinem editum 1927.

† *Monthly Notices*, **76**, 78 (1915).

‡ G. H. Bryan, *Phil. Trans. of the Royal Society, London*, **180**, 187 (1889).

§ *Astrophysical Journal*, **64**, 349 (1926).

$(a - g)$ show an upper limit in eccentricity between 0·96 and 0·97, while the types which are probably spirals seen edgewise show a very strong concentration for eccentricities exceeding 0·90.

The possibility of explaining the spiral arms rests in this case on principles quite different from those suggested by Poincaré. Our point of departure is simply a theorem on the motion of a material particle under gravitation in the equatoreal plane outside of a homogeneous spheroid. In order to make a more general assumption, we may admit that there is, in addition, an arbitrary mass placed at the centre of the spheroid.

Let M be the mass of the spheroid, μM the spherical mass at the centre. The eccentricity of the spheroid being e, the distance from the centre ϖ, we define

$$y = \frac{e}{\varpi}.$$

If θ is the angular co-ordinate in the equatoreal plane, h the area constant, $\varpi^2\frac{d\theta}{dt}$, we get,* taking as unit of length the radius of the spheroid,

$$\left(\frac{dy}{d\theta}\right)^2 + y^2 = \frac{3}{2}\frac{GMe}{h^2}\left[\left(2 - \frac{1}{y^2}\right)\arcsin y + \frac{1}{y}\sqrt{1 - y^2} + \tfrac{4}{3}\mu y\right] + c \qquad (1)$$

where G is the constant of gravitation.

We consider a star moving originally at the edge of the spheroid in a circular orbit. The corresponding value of c is called c_0. We further get in this case

$$h^2 = \frac{3}{2}\frac{GM}{e^3}\left[\arcsin e - e\sqrt{1 - e^2} + \tfrac{2}{3}\mu e^3\right] \qquad (2)$$

We define

$$\left(\frac{dy}{d\theta}\right)^2_{c=c_0} = \psi(y).$$

We then consider a slightly disturbed motion with the same value of h,

$$\left(\frac{dy}{d\theta}\right)^2 = \psi(y) + \Delta c.$$

Because we have by the determinations of h and c_0

$$\psi(e) = 0, \qquad \psi'(e) = 0,$$

the value of $\psi(y) + \Delta c$ in the neighbourhood of $y = e$ is

$$\left(\frac{dy}{d\theta}\right)^2 = \tfrac{1}{2}\psi''(e)(\delta y)^2 + \ldots + \Delta c \qquad (3)$$

When $\psi''(e)$ is negative, the decrease in y for an infinitesimal Δc will be infinitely small before $\left(\frac{dy}{d\theta}\right)^2$ changes sign and a lower limit in y for the

* *Arkiv för Matematik, Astronomi och Fysik,* 19 A, No. 35, p. 7.

moving particle has been attained; but if $\psi''(e)$ is positive, the outward motion will follow a spiral orbit until a limit depending on higher differential quotients has been attained. The value $e = e_1$, where $\psi''(e)$ changes sign and becomes positive, is determined by the equation

$$\frac{e_1 - \frac{1}{2}e_1^3}{\sqrt{1 - e_1^2}} - \arcsin e_1 = \tfrac{1}{6}\mu e_1^3 \quad . \quad . \quad . \quad (4)$$

For $\mu = 0$ this gives

$$e_1 = 0{\cdot}834.$$

For increasing μ the value of e_1 increases but fairly slowly. For $\mu = 0{\cdot}9$ we have $e_1 = 0{\cdot}90$.

When e increases from e_1 to 1 in the case $\mu = 0$, the maximum distance to which the star recedes after an infinitely slight disturbance increases towards infinity. The orbit begins as a close spiral, which for greater distances assumes an elliptic or parabolic shape. When the value of h lies slightly below the critical value (2), the conditions remain qualitatively the same, but a finite value of Δc is needed in order to cause the star to move in the spiral orbit.

A comparison between such orbits and the spiral arms of the nebulæ Messier 33 and 81 was made in the paper cited. It was found that the eccentricity of the attracting spheroid which gives curves similar to the spiral arms exceeds 0·99 in the former nebula and is equal to 0·97 for the latter.* We may observe that these values of the eccentricity lie above the limit of stability for the MacLaurin ellipsoid as well as above Hubble's and Lundmark's observed limit for the elliptic nebulæ.

In what follows we shall try to mould into a more consistent theory the facts which have just been considered. Our treatment here will, however, be very summary.

We do not assume a general development from less flattened to more flattened systems of higher angular speed of rotation. We assume instead that occasionally systems are formed in space, which from the origin are endowed with a very high angular momentum. The only way in which this seems to be possible is by such collisions between systems in which two or more systems get fused together giving up a part of their translatory energy into energy of rotation. We have in the sky several examples of pairs or groups of nebulæ for which we may witness close encounters leading to a definite mixing of the objects. In a system of high angular momentum formed in this way we may assume that a great part of the matter moves nearly in circular orbits around the centre in the "galactic" plane of the system. A great amount of matter must also be assumed to move in a more irregular way in showers of orbits of various inclinations towards the plane of symmetry.

Assuming in the manner of Jeans and Charlier that the frequency function for the element $dx\,dy\,dz\,du\,dv\,dw$ in the steady state ought

* It is probable that Hubble's type Sa corresponds to somewhat lower "effective eccentricities." The absolute lower limit for the development of spiral structure is, of course, the eccentricity 0·834.

to be a function of the energy integral I_1 and the integral of areas on the " galactic " plane I_2, we may consider the value of this frequency function $F(I_1, I_2)$ in a diagram having I_1 and I_2 as axes. We find that the region of possible combinations of I_1 and I_2 in the I_1, I_2 diagram is bounded by the line $I_1 = 0$, indicating the velocity of escape, and by a curve representing the circular orbits in the " galactic " plane. The stars falling near the latter limit will be referred to as constituting the most rapidly rotating " sub-system."

From this sub-system, which can evidently only be present with a great mass in the case of a high angular momentum, the spiral arms are formed by a successive disturbance from ejected matter. The ejection of matter is originated by the sectorial harmonic waves, or by tidal action from exterior bodies.

We shall first consider if there is any action which may produce a uniform rotation of the sub-system in question. In a stellar system of nearly uniform ω and of uniform density in the equatoreal regions there will be a tendency of the stars to gather at certain points and to spread at others. Real concentrations, however, can only occur for the most rapidly rotating sub-system in which the stars are originally nearly at relative rest. If the speed of rotation were not uniform for these condensations, there would, by their interaction at close passages, arise streamings parallel to the " galactic " plane, which would increase the general dispersion of relative velocities in the direction of the radius vector in this plane. When the frequency function of the element $dx\,dy\,dz\,du\,dv\,dw$ is a function of the first integrals I_1 and I_2, however, the dispersion along the radius vector at a certain point in the " galactic " plane must be equal to the dispersion at right angles to this plane. Therefore, in order that a steady state shall be possible, the speed of rotation of the condensations, or " clouds," ought to be uniform. Such a state of motion of the most rapidly rotating sub-system must be rather generally possible by a suitable distribution of the matter moving in circular orbits. We assume here that it is possible for a fairly uniform distribution of density in the " galactic " stratum. The condensations will further be subject to a resistance in their motion due to encounters with members of sub-systems of slower mean speed of rotation, which will cause them to drift towards the centre, thereby depriving them of their isolation. If the speed of rotation ω_{max} of the circular orbits in the plane of symmetry is uniform, we may introduce the term *effective eccentricity*, which is equal to the eccentricity of the homogeneous spheroid of the density ρ of the " galactic " stratum (supposed uniform), which would give the same distribution of central force in the " galactic " plane as the actual stellar system. It is very probable that the central forces in the " galactic " plane *outside of the system* will also in this case be given with a good approximation by the " effective spheroid," at least if we assume, in addition, a certain mass μM at the centre, where μ is at our disposal.

When the effective eccentricity is above the limit of stability for sectorial harmonic waves, matter will be ejected from the sub-system of maximum speed of rotation. The absolute speed of the wave is $\frac{1}{2}\omega$ to begin with, but when matter begins to be definitely detached,

the wave of disturbance is likely to follow the free oscillations of individual particles in the " galactic " plane. The wave of this kind having two opposite elevations at the equator has the absolute speed of rotation equal to zero. The points of ejection ought then to be fixed in space. The successively ejected matter should then, to begin with, follow the same orbits, and a spiral arm defines at first the actual orbit of a star. In several cases it must be possible to consider the development of the spiral arms as a *periodic* phenomenon on account of the asymptotic nature of the orbits.

It may be remarked that the arms evidently wind out from the " mother-system " in the same direction as the rotation of the system, while the opposite is the case according to the suggestion by Poincaré.

The continued disturbance by the ejected matter at the equatoreal boundary may in some cases tend to " roll out " the system almost entirely. The inner spiral arm is the path of this disturbance somewhat expanded by the relative motion during the process of formation. It seems very likely that the inner parts of the arms will get approximately the form of logarithmic spirals, as actually observed. The arm will, in this case, when the central mass successively decreases, no longer define the actual orbit of a single star, but may subsist for long times as the envelope of a great number of individual orbits.*

In the " galactic " regions, where a great amount of matter is moving in nearly relative rest, the white stars of great mass will preferably get formed, while the less rapidly rotating sub-systems of weaker " galactic " concentration will consist of smaller stars of later types, in accordance with the spectroscopic results. The less advanced colour and the greater resolution in photographs taken with big instruments of the spiral arms, when compared with the nuclear regions or with elliptic nebulæ, is thus accounted for.

The white " super-giants " of excessive luminosity will thus not get formed for the less flattened nebulæ, because the necessary conditions for their formation are not present. Neither will such stars be found among the stars having orbits of considerable inclination towards the " galactic " plane in the case of the very flattened systems.

A system of the type considered is never entirely gaseous. It consists eternally of stars, the individual existence of which need not even be destroyed by violent encounters between systems. This does not mean that a certain individual star-molecule has an eternal existence. Especially in the dense " galactic " regions we must assume a steady interchange between stellar and nebulous states of the matter. The process of interchange is, however, of a local and haphazard character.

The general problems of the evolution of the stars have been connected here with conditions of formation depending on the character of the stellar systems in which the stars are members. The further development of the stars may be assumed to proceed parallel with the evolutionary process of the systems. This process must go from very flattened systems to less flattened systems, contrary to the path of evolution according to the " gaseous " theory, and also to the direction

* E. W. Brown, *Astrophysical Journal*, **61**, 97 (1924).

of evolution suggested by Poincaré. The sub-system of highest speed of rotation is likely to loose mass definitely in " hyperbolic " spiral arms upon strong disturbance from the outside, and the interior disturbances, due to local encounters of various kinds, must also tend to break up this sub-system and, in general, cause an advance towards a more chaotic distribution of stellar velocities. This process of evolution may go parallel with a general decrease in the average luminosity of the stars and a shift towards a more advanced mean spectral type.

Hubble has made probable that the average length of the polar axis is nearly the same for different kinds of elliptic nebulæ and for the spirals. This suggests, roughly speaking, that the elliptic nebulæ may be identical with spiral nebulæ stripped of the arms. Hubble seems to accept the course of development suggested by Jeans in the form of an *expansion* in the equatoreal plane, without attempting to account for the successive increase in angular momentum necessary for such a process.

The present theory connects a multitude of facts concerning our stellar system and the spirals, which have only been touched upon very incompletely in this exposition of the general principles. With full appreciation of the beautiful theory of Jeans, I have therefore ventured to give the present account of the theory which treats the problems from an older, though considerably modified, point of view.

Distances of the Galactic Cepheids, Magellanic Clouds, and Globular Clusters. By C. D. Perrine.

Recently direct evidence has been encountered which contradicts the high luminosities obtained for the Cepheids and stars of early spectral type in the Galactic System. A possible source of error in the determination of these distances has been suspected for some time and has now been confirmed. The object of the present paper is to give the preliminary results of these investigations.

The Globular Clusters and Magellanic Clouds.

In the Globular Clusters the brighter non-variable stars have a small dispersion and are essentially of F (and perhaps early G) type. This condition has been found in so many that it may be taken as representative, and is so well known and accepted that it needs no further discussion here.

In the two Magellanic Clouds are a considerable number of B- and O-type stars whose magnitudes show small dispersions, particularly in the Small Cloud. These stars set a fairly definite *upper limit* to the brightness of such spectral types in those agglomerations.

A comparison of these stars with the apparent magnitude of Cepheids

in the same agglomerations, in accordance with the apparently definite functioning of the Period-Luminosity Law discovered by Miss Leavitt, gives at once the relative brightness of O, B, and F-type stars in the Magellanic Clouds and Globular Clusters, referred to Cepheid variables of definite periods.

If now we may extend the universality, which is generally accepted, of the Period-Luminosity Law to include such brightness relations to spectral type as the above, we can reverse the usual practice, and by means of the relations observed in the Globular Clusters and Clouds which are free from uncertainties of relative distance, obtain the relations which exist in our Galactic System. And, finally, the absolute magnitudes of stars of spectral types O, B, and F in our system may be obtained by comparison with the absolute magnitude of the Sun, which is well established. Absolute magnitudes of the Galactic Cepheids may also be obtained with somewhat wider margins of uncertainty from the absolute magnitudes of non-variables which have been obtained in considerable numbers from observed parallactic displacements.

This procedure rests on the similarity of stellar evolutionary processes and spectral types in all parts of the universe. Such an assumption is explicitly made by some investigators and tacitly by others. The dependence of absolute luminosity upon spectral class (at least the three in question) receives support from the behaviour of the stars in the Globular Clusters and Magellanic Clouds, where it appears to be just as evident as the Period-Luminosity Law. I have encountered evidence of the same dependence among the B-class stars of our Galactic System. On the definite evidence in the Globular Clusters, Magellanic Clouds and Galactic System, and in the absence of contrary evidence, the assumption of a general dependence of luminosity upon spectral class will be made for the purpose of this comparison. The numerical results are given in Table I.

TABLE I.

Differences of Absolute Magnitudes in the Galactic System and Magellanic Clouds.

Non-variable Brightest Stars.	Globular Clusters (7).	Small Cloud.	Large Cloud.
Spectrum	F6	B and O	O
Photo. mag.	+13·8	+11·2 (10)	+10·8 (9)
Cepheids (P=6^d·0) (median mag.)	+13·1 (328)	+15·9 (106)	+15·7
Brightest—Cepheids	+ 0·7	− 4·7	− 4·9
Galactic Stars for Comparison.			
Brightest F6–Go	− 0·5		
,, B5–Go	..	− 4 1	
,, O –Go	..	..	− 4·4

The results for the Globular Clusters were taken from Shapley's paper on these bodies.*

* *Astrophysical Journal*, **48**, 89, 1918.

The median magnitude, 15·9, of the Cepheids in the Small Cloud was taken from *Harvard Circular*, No. 280. The median magnitude of the Cepheids in the clusters, 13·1, is that derived from Shapley's results corrected to a period of $6^d{\cdot}0$ by applying the difference of −2·0 from his Period-Luminosity curve.

The Cepheid magnitude for the Large Cloud was derived from a comparison of 250 variables taken at random in each cloud from Miss Leavitt's tables in *Harvard Annals*, vol. 60, part iv, upon the assumption of similar periods, the only course open to me.

The magnitudes of the B and O stars in the Small and Large Clouds are from the Draper Catalogue and photographs taken here, all on the Harvard system.

The galactic results are from Plate VI. accompanying H. D. Curtis's article "Stellar Luminosities." * This diagram was derived from directly measured parallaxes by photography and the spectroscopic method, and represents over 2000 stars.

Leaving out of consideration for the present the giant branch of the curve, two things attract the attention :

A. The change of brightness with spectral type is a curve, the rapid change in K and M types becoming practically zero at B and O.

B. Preference for a dispersion of 2 or 2½ magnitudes.

I have drawn a curve through the densest portions, the co-ordinates of which are as follows :—

Spectral Type.	Absolute Magnitude.	Spectral Type.	Absolute Magnitude.
Bo	+0·9	Go	+4·3
B5	1·2	G5	5·4
Ao	1·6	Ko	6·4
A5	2·0	K5	7·6
Fo	2·6	Ma	+9·4
F5	+3·4		

For comparison with the Clusters and Clouds I have chosen the type Go as being at once near the Sun, whose absolute magnitude is accurately known, and the Cepheid variables of moderate periods. For comparison the galactic B and O stars were corrected by − 1·0 magnitude to make them comparable with the Clouds, in particular where it was found that, in the case of the O-type stars in the Large Cloud, there was also a dispersion of 2 to 2½ magnitudes—the brightest ones used in the Table I. being about one magnitude brighter than the average.

No refinements have been attempted, as the data and the state of our knowledge do not warrant it. Outstanding facts and approximate numerical results only have been aimed at. In my opinion the absolute magnitudes based upon the parallactic displacements are certainly too bright, and on the average perhaps as much as a magnitude for Go and earlier. That opinion is based upon the consideration that

* *Publications A. S. P.*, **34**, 33, 1922.

no measured parallax can be too large, but is certain to be too small by an unknown amount. If the Sun is an *average* Go-type star, then such a conclusion is corroborated by the fact that the Go absolute magnitude derived from the foregoing curve is 1·0 ± brighter than the accurately known magnitude of the Sun.

In Table I. the discordance of the bright non-variable F-type stars in the clusters is noticeable. Until the underlying relationships, however, are better known, it is not deemed advisable to lay much stress upon it.

As the B and O stars in both the Large and Small Cloud agree in showing about the same relation of absolute brightness to F- and G-type stars as in the Galactic System, we may assume sufficient similarity of behaviour to obtain the distances of Clusters and Clouds through a comparison of the brightest stars of those aggregations with the absolute magnitudes of similar spectral types in the Galactic System as given by the Curtis curve, entirely independent of the Cepheids or the Period-Luminosity curve. As the galactic absolute magnitudes depend upon the observed parallaxes of over 2000 stars, they may be said to be the strongest data of the kind which we have.

The procedure is very direct, involving only the spectral types and the apparent magnitudes of the stars in the Clouds and Clusters to be compared with the absolute magnitudes of similar types in the Galactic System, which have been obtained directly from their apparent magnitudes and observed parallaxes. In its fundamentals it is essentially the same as through the Period-Luminosity curve of the Cepheids—the only difference being that different groups and types of spectra are used, of non-variables.

Using data from Table I. we arrive at the results given in Table II.

Table II.

Distances derived from Brightest Stars (Non-variables).

	Globular Clusters.	Small Cloud.	Large Cloud.
Mean photo. mag. brightest stars	+13·8	+11·2	+10·8
Absolute photo. mag. galactic stars same spectral type	+ 3·6	+ 0·9	+ 0·9
Difference	10·2	10·3	9·9
Relative distance	110	115	96
Parallax	″·00091	″·00087	″·00104
Distance in light years	3600	3800	3100
Spectral type	F6	B and O	O

These distances do not all agree with those derived by the use of the Period-Luminosity Law, although derived from essentially similar data except the stars which are not variables. These distances are approximately only $\frac{1}{30}$ those given by the Cepheids. Instead of 105,000 light years for the Small Cloud as derived by Shapley * from the Cep-

* *Harvard Circular*, No. 280.

heids and the Period-Luminosity Law, the B and O stars give only 3800 light years. These B and O stars give a difference due to distance of 10 magnitudes where Shapley obtains 17 + by the use of the Period-Luminosity curve. This evidence indicates a correction to Shapley's curve of 7 magnitudes.

The amount and consistency of the data for the non-variables, as well as the greater directness and freedom from assumptions, give confidence in the general correctness of the results derived from them. On the other hand, various inconsistencies and improbabilities have thrown doubt on the small parallaxes obtained for the 11 basic Galactic Cepheids.

Now that the asymmetry which has been known to exist in the parallactic motions of the B-class stars has been established in the Galactic Cepheids also, results from these latter stars are seen to be invalidated.

Asymmetry in Parallactic Motions.

This asymmetry was discovered in the proper motions of the B-class stars some years ago and a short paper prepared but not printed. It was, however, referred to in an article on the solar apex.* Briefly stated, it was that the proper motions in the northern sky were only about half as large as those in the southern sky. This in connection with the strong Galactic preference of the B stars and the direction of solar motion indicated differential motion on opposite sides of the axis of solar motion in the plane of the galaxy, the Sun moving with the northern stars.

The subject was dropped until more, and especially stronger, southern positions became available. The especial importance of this asymmetry in the determination of distances of the Cepheids has led to a re-examination, this time with respect to the v component of solar motion.

If there is a skewness of this nature in the general motions of the northern and southern portions of the Galaxy, then distances derived from the northern stars will be *too large*, parallaxes too small. And here is the weakness in the distances derived for the stars used by Shapley as the basis of his Period-Luminosity work. Not only is the number very small to use for an investigation of such proportions, but, what turns out to be most important, these 11 stars are confined essentially to the *northern portion of the Galaxy*. The extreme southern portion is wholly unrepresented. Shapley states with regard to the distribution of these 11 Cepheids—" . . . their distribution in Galactic longitude is satisfactory." † Yet, according to his own statistics, Galactic longitudes from 164° to 329° are unrepresented. And it is in this region, which is 165° or nearly half the circumference, where the skewness in parallactic motion is revealed so strongly by comparison.

Asymmetry is present not only in the B stars, but is now shown by southern stars to exist also in the Cepheids. An examination of the

* *Astrophysical Journal*, **45**, 107, 1917.
† *Ibid.*, **48**, 95, 1918.

P.G.C. shows it to be present at least among the stars of small p.m. It also manifests itself in the discordances in δ of the apex of solar motion derived from proper motions, and appears to be related to the systematic corrections to many of the early meridian place catalogues. It is under more detailed investigation with especial reference to these last two problems and the general motions of the Galactic System.

TABLE III.

B-Class Stars—Northern.

R.H.P. No.	h m	° ′	Vis. Mag.	Obs. V. km./sec.	V_1. km./sec.	τ. ″	υ. ″
8143	21 13·5	+38 59	4·3	− 7	+ 4·6	+·006	+·005
8146	13·8	34 29	4·4	+11	+22·4	+ 18	− 23
8238	27·4	70 7	3·3	− 5	+ 4·9	0	− 12
8279	35·2	61 38	4·9	−12·5	− 2·1	+ 13	− 48
8335	43·1	48 51	4·3	−19·2	− 8·7	+ 3	− 4
8522	22 16·7	27 50	4·9	+ 9	+16·5	0	− 5
8523	16·9	46 2	4·7	− 9·0	0	− 2	− 20
8541	20·4	48 58	4·6	−25·7	−16·8	+ 12	− 7
8762	57·3	41 47	3·6	−16	− 9·4	+ 14	− 27
8797	23 2·4	58 53	4·9	− 2·0	+ 5·6	− 7	− 10
8926	25·4	58 0	4·9	−14·8	− 8·2	− 17	− 22
9071	53·9	55 12	4·9	− 4	+ 1·2	+ 2	− 12
153	0 31·4	53 21	3·7	+ 3·0	+ 6·4	− 1	− 23
154	31·5	33 10	4·4	+ 8·8	+ 8·6	− 2	− 23
226	44·3	40 32	4·4	−23·9	−23·4	+ 9	− 27
264	50·7	60 11	2·2	− 3	+ 0·6	− 11	− 29
335	1 3·7	46 42	4·3	− 4	− 3·3	− 32	− 65
477	34·7	40 4	4·9	− 2	− 0·3	+ 11	− 27
496	37·4	50 1	4·2	0	− 0·2	− 2	− 33
542	47·2	63 11	3·4	− 9	− 6·7	− 2	− 42
936	3 1·7	40 34	2·3	+ 4·1	− 2·2	− 3	− 9
1034	20·9	48 43	4·9	+ 8	+ 3·5	− 10	− 30
1035	21·0	59 36	4·4	− 5	− 6·2	− 3	− 3
1040	21·9	58 32	4·8	− 6	− 7·6	− 9	− 6
1123	36·0	33 39	5·0	+ 3	− 6·7	− 3	− 15
1131	38·0	31 58	3·9	+18·5	+ 8·2	+ 4	− 26
1203	47·8	31 35	2·9	+20·4	+ 9·7	− 3	− 21
1204	48·6	+62 47	4·9	+ 5	+ 4·1	− 5	− 1
			4·22	9·2	7·1	−·0007	−·0196

TABLE IV.

B-Class Stars—Southern.

R.H.P. No.	h	m	°	′	Vis. Mag.	Obs. V. km./sec.	V_1. km./sec.	τ. ″	v ″
3642	9	4·8	−70	8	4·9	+26	+ 6·3	+·001	−·008
3654		7·4	44	27	5·0	+35	+13·7	− 1	− 14
3659		8·4	58	33	3·6	+23·3	+ 2·5	− 19	− 36
3663		9·0	61	54	4·2	+16	− 4·5	− 14	− 45
3734		19·0	54	35	2·6	+21·9	+ 1·2	− 10	− 19
3825		31·5	58	47	4·2	+21·7	+ 1·6	+ 7	− 16
3856		36·6	60	53	4·7	+26	+ 6·2	+ 9	− 38
3940		53·4	54	6	3·7	+14·4	− 5·1	− 8	− 21
4074	10	17·2	55	33	4·6	+ 9	− 9·6	− 1	− 18
4118		25·0	30	5	5·6	+25	+ 8·4	0	− 45
4140		28·5	61	11	3·6	+24	+ 5·8	+ 5	− 25
4198		38·9	58	42	5·4	0	−17·8	− 20	0
4199		39·4	63	52	3·0	+16	− 1·8	+ 8	− 23
4234		44·8	80	1	4·6	+22	+ 4·8	− 15	− 46
4460	11	30·0	53	42	4·8	+12	− 3·5	+ 12	− 64
4467		31·2	62	28	3·3	+11	− 5·1	− 15	− 45
4552		47·8	33	21	4·4	−20	−32·0	+ 18	− 53
4618	12	2·9	50	6	4·8	+16·4	+ 2·8	+ 5	− 29
4638		6·4	51	48	4·2	+28	+14·3	− 5	− 49
4656		9·8	58	12	3·1	+25	+10·7	− 4	− 47
4679		13·0	63	26	4·3	+19·9	+ 5·1	− 5	− 50
4730		21·0	62	33	1·6	+ 7	− 7·4	− 18	− 43
4798		31·2	68	35	2·9	+13·5	− 1·3	− 8	− 41
4817		34·4	39	26	4·8	+13	+ 2·8	− 16	− 61
4823		36·2	59	8	5·0	+19·5	+ 6·1	− 2	− 25
4848		40·6	55	56	4·9	+18·2	+ 5·4	− 20	− 51
4853		41·9	59	9	1·5	+13	− 0·2	− 5	− 56
4898		48·8	56	38	4·3	+13·2	+ 0·7	+ 3	− 36
4899		48·8	56	37	5·5	+ 7	− 5·5	− 6	− 26
5132	13	33·5	52	57	2·6	+ 6	− 4·0	− 4	− 41
5190		43·5	41	11	3·5	+ 6	− 0·9	+ 2	− 42
5193		43·6	41	59	3·3	+12·6	+ 5·5	− 1	− 28
5210		46·0	32	30	4·7	+14·1	+ 9·4	− 1	− 60
5221		47·5	31	26	4·8	+ 8	+ 3·7	− 5	− 28
5248		52·2	41	36	4·0	+ 5·3	− 1·2	+ 2	− 34
5249		52·5	44	19	4·2	+ 4	− 3·2	− 3	− 46
5267		56·8	59	53	0·9	+12	+ 1·3	− 8	− 40

TABLE IV.—*continued.*

R.H.P. No.			Vis. Mag.	Obs. V.	V_1.	τ.	v.
	h m	° ′		km./sec.	km./sec.	″	″
5285	13 59·9	40 42	4·5	+12·4	+ 6·5	−·013	−·036
5358	14 13·3	55 56	4·4	+ 3·5	− 5·7	+ 1	− 32
5378	16·8	39 3	4·6	+ 9·2	+ 4·5	− 9	− 48
5395	19·7	44 46	4·6	−16·6	−22·7	− 14	− 28
5425	25·9	50 1	4·6	− 0·6	− 7·8	+ 28	− 46
5440	29·2	41 43	2·6	0	− 4·8	0	− 49
5469	35·3	46 58	2·9	+ 8	+ 2·0	− 3	− 36
5528	45·1	43 9	4·5	+ 7·2	+ 2·7	+ 4	− 50
5571	52·0	42 44	2·8	0	− 4·1	+ 4	− 69
5576	52·7	41 42	3·4	+10	+ 6·2	− 7	− 35
5687	15 12·4	40 26	5·8	+21	+18·3	+ 1	− 41
5708	15·9	44 20	3·7	+14	+10·3	0	− 28
5781	29·0	44 37	4·8	+ 5	+ 1·7	− 16	− 36
5839	36·3	34 23	4·8	+12	+12·0	+ 8	− 48
5928	50·7	28 55	4·0	+ 3·2	+ 5·4	− 4	− 33
5967	56·8	−38 19	5·0	+ 5	+ 4·5	+ 7	− 43
			4·04	..	6·4	−·0029	−·0379

The data for the individual stars of class B were taken from the *L.O. Bulletin*, No. 195, and are given in Tables III. and IV. They depend upon the P.G.C. of Boss.

The comparison between northern and southern stars is given in Table V.

TABLE V.

B-Class Stars.

Limits of Right Ascension.	No. of Stars.	Vis. Mag.	V_1.	τ.	v.
			km.	″	″
Northern, 21^h to 4^h	28	4·22	7·1	−·0007	−·0196
Southern, 9^h to 16^h	53	4·04	6·4	−·0029	−·0379

Making use of Gould's General and Zone Catalogues in connection with the Albany, Cape, and Cordoba Catalogues for 1900 and the La Plata Catalogues for 1925, I have determined the proper motions of 24 southern Cepheids. The individual results are given in Table VI., and comparison with the data for the 11 which Shapley used as a basis for his work,* in Table VII.

The positions were reduced to the system of the P.G.C. by the application of Boss's corrections.

* *Astrophysical Journal*, **48**, 94, 1918.

TABLE VI.

Southern Cepheid Variables.

Catalogue. A.G.C.	Catalogue. V.J.S.	Mag. Max.	Mag. Min.	Spec.	Period.	α. 1900·0.	δ. 1900·0.	μ. α.	μ. δ.	μ.	τ.	v.
					d	h m	° ′	s	″	″	″	″
10,946	2139	7·0	8·5	K*op*	41·3	8 9·2	−34 16·6	+·0004	−·060	·060	−·060	+·006
11,451	2145	7·4	8·1	G5	6·7	8 26·7	59 47·3	− 14	− 020	·023	− 23	+ 3
11,634	2147	7·5	8·6	Ko	Kurz	8 33·4	43 45·9	− 44	− 34	·059	− 43	− 40
11,668	2148	7·7	8·5	G5	4·6	8 34·4	47 0·7	− 6	− 55	·055	− 54	+ 9
12,793	2166	7·5	8·2	G5	4·4	9 19·2	55 32·0	+ 8	− 18	·020	− 16	+ 13
13,336	2181	3·6	4·8	G*op*	35·5	9 42·5	62 2·8	− 40	− 12	·030	− 19	− 23
14,701	2282	7·8	9·2	G5	19·0	10 40·6	57 2·4	+ 31	− 40	·047	− 41	+ 23
15,004	2333	6·8	8·0	Go	38·7	10 53·7	59 11·8	− 15	0	·012	+ 1	− 12
16,671	2403	6·4	7·3	F8*p*	9·7	12 7·4	69 35·7	− 26	− 6	·015	− 3	− 15
16,832	2408	6·8	7·6	Go	6·7	12 15·9	61 43·6	− 30	0	·021	+ 8	− 20
16,882	2411	6·8	7·9	F5	5·8	12 18·1	61 4·5	− 18	− 40	·042	− 33	− 26
17,263	2425	6·7	7·6	G5	0·9	12 36·0	68 51·5	− 15	+ 24	·025	+ 25	+ 1
17,531	2434	6·5	7·6	F5	4·7	12 48·4	57 53·3	+ 2	− 22	·022	− 21	− 8
19,641	2509	6·4	7·8	F5	5·5	14 25·4	56 26·7	− 12	− 29	·031	− 13	− 28
20,693	2528	6·7	7·4	F5	3·4	15 10·8	66 7·7	− 45	− 19	·033	+ 10	− 31
21,614	2550	6·4	7·4	G5	6·3	15 52·2	63 29·5	− 36	− 18	·030	+ 13	− 27
21,762	2559	7·7	8·4	F5	2·6	15 58·4	62 38·3	− 15	− 49	·050	− 13	− 48
22,031	2573	6·4	7·6	G*op*	9·8	16 10·6	57 39·2	− 37	− 25	·039	− 18	− 35
22,956	2645	6·9	8·0	F5	6·1	16 51·8	33 27·2	− 29	− 15	·039	+ 30	− 24
24,120	2682	4·4	5·0	F9*p*	7·0	17 41·3	27 47·6	+ 17	− 9	·025	− 24	− 7
24,212	2685	7·5	9·0	G5	20·3	17 44·3	33 40·5	− 14	− 16	·024	+ 17	− 17
24,306	2691	7·5	8·2	Ao	0·9	17 48·6	34 12·9	+ 48	− 29	·067	− 62	− 26
24,577	2711	4·3	5·1	G*op*	7·6	17 58·6	29 35·1	+ 36	− 50	·069	− 48	− 50
24,788	2733	6·8	7·4	F5	5·1	18 7·0	−23 8·5	− 26	− 48	·060	+ 38	− 47
		6·65	7·63	..	10·8	..	..	..	..	..	−·015	−·018
										Omitting first seven	−·005	−·025
										First seven	−·037	−·001

After deriving my proper motions for the southern Cepheids I became aware of Wilson's work,* which had escaped my notice owing to my absence. I have taken his results in the same southern (Galactic) region, which are for the greater part the same stars I used, and his results are given at the bottom of Table VII. as well as the adopted values of τ and v. Wilson for the most part used different recent observations from mine, and about a third are different stars. None of this data appears to be sufficiently strong to justify further discus-

* *Astronomical Journal*, No. 821, **35**, 35, 1923.

sion, especially as these and other southern Cepheids will be observed here at the first opportunity, to obtain strong places.

TABLE VII.

Cepheid Variables.

Asymmetry of Parallactic Motions.

Limits of Right Ascension.	No. of Stars.	Vis. Mag. Median.	Period.	V_1.	τ.	v.	v'.
			d	km.	″	″	″
Northern, $17^h\ 41^m$ to $6^h\ 58^m$	11	4·75	6·0	8·4	−·0020	−·0157	
	10 *	5·02	6·2	8·8	−·0006	−·0132	
Southern, 8^h to 19^h	24	7·14	10·8	..	−·015	−·018	−·057
	17 †	..	..	..	−·005	−·025	−·080
	7 ‡	..	..	..	−·037	−·001	
F-type stars §	159	4±	..	13·9			
Southern (Wilson) 8^h to 19^h	34	7·2	..	..	−·006	−·012	−·038
	25 †	..	..	..	−·004	−·014	−·045
	7 ‡	..	..	..	−·012	·000	
Southern, adopted	All †	..	..	..	−·010	−·015	−·048
					−·004	−·019	−·061

As most of my proper motions are based on longer intervals than Wilson's, they probably are entitled to greater weight. But in view of other factors of uncertainty, it seems sufficient to give them equal weight, which I have done.

Considering the data, the agreement of our results is probably as good as can be expected. In one respect it is notable, that of the behaviour of the first seven stars in my list, mentioned in the next paragraph.

These first seven stars of my list show a marked discordance in their components of solar motion. Their *mean v component is nil* and their average τ *component consistently large*, and of the same sign. The same seven stars of Wilson's list also show no v component and a τ component much smaller than mine, but of fairly consistent sign. Wilson has two stars in that region which I have not, which tend to confirm the explanation given below of this peculiar behaviour, which appears to have a bearing on the underlying cause of the small parallactic motions and displacements of the northern Cepheids.

This explanation was suspected from my list. It is that we are dealing with stars of two different streams, one having a general relative motion in the direction of the solar motion axis, and the other stream nearly at right angles, probably in the general direction of the

* Omitting α Urs. Min.
† Omitting 7 stars with no sensible v components.
‡ Seven stars with no sensible v components.
§ Radial velocities of F-type non-variables added for comparison.

"ellipsoidal axis." The same preference for two different motions, in varying proportions, has been noted elsewhere, and in various investigations. If this hypothesis is correct, then we have a logical explanation of the cause of the asymmetry under discussion and of the very small parallactic motions and displacements of the northern Cepheids.

As has been stated, these results are all on the P.G.C. system of Boss. Presumably few or none have had the small corrections to that system, which were given by Boss on p. 28 of the P.G.C., applied. An examination shows that these corrections would *decrease* the asymmetry by from 5 to 10 per cent. Raymond's corrections to $\mu\delta$ * would have a small effect of the same nature at the beginning and end of the southern list, but not enough materially to affect the results.

In view of the results which will soon be available from the discussion of modern observations in the southern sky, it does not seem necessary to do more than determine the order of these corrections. An examination of Zimmer's positions indicates that any corrections to the P.G.C. will be too small materially to affect the asymmetry found.

We may use these results, therefore, with all the confidence which the most modern meridian circle positions inspire in so far as the reality and general magnitude of this asymmetry are concerned, and only limited perhaps by the question of sufficiency of data. Asymmetry appears to be as definitely established by the existing data as are other interpretations of the same data—as, for example, star-streaming or even solar motion itself.

The asymmetry in the case of the B stars (Table V.) is evident at a glance. It is not evident in the case of the Cepheids (Table VII.) until the *differences of magnitude and period of the groups* have been taken into account. And here is the omission in Wilson's work which invalidates his conclusions. This will be referred to later.

The relationships connecting period and luminosity in the Cepheids permit, and require before data is comparable, that they be made homogeneous in these respects. The magnitudes of the two groups of B stars differ by less than 0·2, which may be considered negligible. The Cepheids of Table VII., however, differ both in magnitude and period—the magnitudes are 4·75 and 7·14 and the periods $6^d{\cdot}0$ and $10^d{\cdot}8$, respectively for the northern and southern groups. If the v components of the southern groups are corrected by means of Shapley's Period-Luminosity curves to the northern group we have the values of v' given in the last column of Table VII.

This comparison of Cepheids in two opposite regions of the Galaxy shows, therefore, the same asymmetry as the B-class stars of the P.G.C., and this is manifested in other ways.

The underlying cause of this asymmetry is undoubtedly connected with the phenomena of Kapteyn's two streams, but beyond its effect on the distances of these Cepheids, does not concern the present investigation. The essential point is that with differences of stream motion on opposite sides of the solar motion axis, the method of getting

* *Astronomical Journal*, No. 857–8, **36**, 136, 1926.

distances by using parallactic motion in combination with an arbitrary solar velocity is obviously invalidated. We do not know which side is correct or whether either one is. Neither do we know what value of the solar velocity to use, for that has been determined from stars in an entirely different part of the sky. And it now looks as though the usually accepted solar velocity had been derived essentially from a stream of rapidly moving stars and that the Cepheids belonged to a stream which had little or no motion in the direction usually assumed for the Sun, and therefore little parallactic displacement, at least for the northern group. Indeed if we make an assumption of a very small solar velocity the distances and magnitudes of these Galactic Cepheids would agree satisfactorily with the results derived from non-variable stars, the Magellanic Clouds and Globular Clusters.

An examination of the individual radial velocities of these 11 Cepheids shows that 3 (P.G.C., Nos. 325, 637, and 5807) are rather closely grouped in the far north and their velocities are satisfactorily reduced by the application of the usual solar motion correction. The remaining 8, however, including the 2 largest, P.G.C. 1629 and 4564, are better represented by assuming a preferential motion of 20 km. towards an ellipsoidal axis at 6^h, + 15°, the means being 10·2 corrected for the usual solar motion as against 8·5 corrected for motion towards 6^h, + 15°. Two of these 8 are not improved but made much worse by either system of corrections, and 2 others are small or but little improved. It is necessary to assume, therefore, that all of these 4 are travelling with the Sun rather than with the system giving the usual solar velocity. Upon such an assumption for these 4 stars, if we reject α Urs. Min. and δ Cephei as belonging to the solar motion stream and correct the remaining 5 for preferential motion towards 6^h, + 15°, the average residual velocity for the 9 stars becomes 4·8 km.

To show the impossibility of using the method of parallactic displacements when such asymmetry exists it is only necessary to point out that had Shapley based his Period-Luminosity curve on this group of southern Cepheids instead of his northern one, he would have arrived at a result 3 *magnitudes fainter*. In other words, his distances would have been only *one-fourth* of those he found from the northern stars. This is without taking into consideration the preferential motion of his group which he found and which would have tended still further to reduce his distances. But this result is just as meaningless as that from the northern group without independent corroboration.

It must be emphasised that the *mean* would not eliminate the uncertainty. For this is one of the cases where a knowledge of the underlying conditions is necessary before any attempt can be made to interpret and use numerically the observed data.

The same considerations make it obvious that parallactic *displacements* are likewise invalidated until such time as these motions are better understood. Hence there appears to be no alternative under the circumstances to rejecting *in toto* all of the distances and absolute magnitudes derived from the Galactic Cepheids by the use of their observed parallactic motions in combination with any of the usually accepted solar velocities.

If it should be argued that parallaxes of 0″·05 or 0″·06 for the northern Cepheids are impossibly large, it would be necessary to produce extremely strong evidence to establish such a contention.

Referring to Wilson's investigation—" From this and the considerations outlined above, we conclude that the most *probable correction* to Shapley's parallaxes indicated by the data presented in this paper is an *increase of approximately* 30 per cent.*

When the differences of magnitude and period are taken into account this increase becomes 4 to 5 *times* instead of 30 per cent.

The mean magnitude of Wilson's stars is 7·6 and the period $9^{d}\cdot1$. When reduced to the same period as Shapley's 11 northern Cepheids, Wilson's stars are effectively 3 magnitudes fainter than Shapley's. Now by assumption this difference of brightness can only be ascribed to greater distance. It cannot be ascribed to difference of intrinsic luminosity, and there is no known reason justifying an assumption of selective absorption or other local condition in the southern region to account for the difference. And there is known from much other evidence to be just such an asymmetry as is indicated by ascribing the difference to distance. In fact if asymmetry did *not* appear in the Cepheids, that fact would of itself require explanation.

Kapteyn and van Rhijn from their investigation concluded that Shapley's parallaxes for the Galactic Cepheids needed to be increased 7·6 times.† The present investigation of asymmetry in the parallactic motions of these Cepheids does not profess to establish a definite value of those parallaxes, but shows that Shapley's values must be *at least* 4 or 5 times too small, and that Kapteyn's and van Rhijn's value may even be conservative. If the comparison of the B, O, and F non-variables prove to be as reliable as they now appear, even their factor of 7·6 may need to be doubled or trebled.

The parallaxes and distances deduced in this paper from the non-variable stars of O, B, and F types are admittedly tentative. Several considerations tend to show, however, that the true distances of those objects are not as much as twice those given in Table II., that those distances are of the right order.

This method of using the non-variables as well as that of working backward from the Clouds and Clusters to our Galactic System through the Cepheids and obtaining all distances relative to our Sun, is being more fully investigated and will be made the subject of a later paper. It may be said, however, that for the two Magellanic Clouds the distances derived with reference to the Sun through the Cepheids are of the same order as those found above from the non-variable B and O stars, being 3300 and 4100 L/Y for the Large and Small Clouds respectively.

Similar procedure for N.G.C. 6822 gives 22000 L/Y, and for the Spiral Nebula M33 30000 L/Y.

The above distances locate all of these objects as members of the Galactic System.

* *Astronomical Journal*, **35**, 44, 1923.
† *Bulletin Ast. Inst. of the Netherlands*, No. 8, 1922.

Summary.

The results of this investigation may be summarised by saying that a strongly marked asymmetry exists in the parallactic motions of the Galactic Cepheids, which renders distances derived from them meaningless. As there is no known way to eliminate this skewness, nor to determine the corresponding solar velocities to use with the observed parallactic motions, this method of obtaining distances is invalidated for the present.

Distances of the Magellanic Clouds and Globular Clusters obtained through non-variable stars of O, B, and F types are of the order of 4000 light years.

A number of consequences follow the failure of small parallactic motions to give true distances, and from the evidence afforded by non-variables in the Magellanic Clouds and Globular Clusters. The most sweeping, perhaps, is that among the early-type stars at least there is no acceptable evidence of a greater brightness than absolute magnitude 0.

Others are :—

The Galactic Stars of early type are only 3 or 4 magnitudes brighter than the Sun (and stars of F and G type).

The absolute magnitudes of the Cepheids are of the order of our Sun.

The so-called " eclipsing variables " are comparable in brightness with other stars of their respective spectral classes.

The peculiar variations in brightness of the Cepheids and Algol stars are not due to great mass or size.

The densities of the Cepheids and Algol stars are of the same order as that of the Sun.

The Globular Clusters, the Magellanic Clouds, and some evidence in our Galactic System, particularly among the B-class stars and the Cepheid Variables, point to restricted dispersions in brightness among stars of middle and early spectral type.

Observatorio Nacional Argentino,
Cordoba : 1926 *December* 3.

Note on Photographic Magnitude Equation. By H. Knox-Shaw, M.A., D.Sc., Radcliffe Observer.

Photographic magnitude equation is no new phenomenon. Its possible existence was recognised in the early days of astronomical photography for position, but I have been unable to find any account of a detailed study of a systematic error of this nature. So I have thought that the description of a systematic magnitude equation, which has come to light in the course of the proper motion work now being carried out with the 24-inch refractor at the Radcliffe Observatory, may be of interest.

Guiding error is usually and rightly regarded as a frequent cause of magnitude equation, but it is from its nature likely to differ in its effect from night to night and from plate to plate, and to vanish for the mean of a number of plates; and it operates only in right ascension, for any drift in declination due to refraction or maladjustment of the polar axis, being uniform and continuous in its action, cannot cause it. It was therefore somewhat surprising, on analysing the proper motions in declination of the first ten of the Selected Areas to be completed here, to find a systematic difference of proper motion depending on magnitude. Each of the ten areas showed practically the same effect. The existence of a similar effect in the proper motions in right ascension was not at first obvious, as owing probably to the presence of a certain amount of guiding error there was greater variation from plate to plate, but the mean of the ten areas showed a similar systematic difference with magnitude. The mean results from these ten areas, corrected for parallactic motion, and made absolute by applying the parallactic motion of the comparison stars, which were on an average 86 in number and of mean magnitude 13·5, are given in Table I.

TABLE I.

Mean Absolute Proper Motions in Ten Selected Areas, freed from Parallactic Motion.

Int. Mag.	No. of Obs.	Mean Proper Motions (unit 0″·001/annum).	
		R.A.	Dec.
B< 9·5	16	+37	−25
9·5	21	+16	−25
10·0	21	+10	−15
10·5	30	+13	−14
11·0	40	+ 3	−18
11·5	95	+ 3	− 9
12·0	117	+ 2	− 4
12·5	161	0	− 3
13·0	277	0	0
13·5	300	0	+ 2
14·0	456	− 1	+ 1
14·5	547	− 1	+ 4
15·0	628	− 1	+ 1

The question at once arose as to how much, if any, of this could be real. The ten areas in question are Nos. 30, 34, 36, 59, 60, 61, 63, 91, 104, 110; three of them are thus in declination + 45°, four in + 30°, one in + 15°, and two on the equator. Owing to the southerly declinations of both the drift apices the position-angles of the apex of Drift II. from these fields are all markedly southerly, and where those of Drift I. are northerly, the effect of this drift is considerably discounted by fore-

shortening. So one would expect a resultant southerly drift of the stars in all these areas. This should show itself as a southerly proper motion of the bright stars relatively to the faint stars, increasing with the brightness of the stars, for if a number of stars are moving in the same direction with the same velocity, the brighter stars, being on the average the nearer, will have the larger proper motions. But the effect found seemed too large, and a simple calculation at once showed that this was so. If V kilometres a second be the component in a given direction of the velocity of a star of parallax π and proper motion μ in the same direction, then

$$V = 4{\cdot}74\frac{\mu}{\pi},$$

and taking the average parallax of a star of magnitude 9·5 as 0″·006, we obtain V = 19 km./sec., which is clearly too large. Moreover, the proper motions in right ascension cannot be accounted for at all in this way.

Thus it seems certain that a large part of the effect is an error. The above argument, however, shows that it is not sound to assume that the whole of the effect is error, and so to determine its amount from the mean of a number of plates, unless these plates are so distributed that it is possible to separate the effects of any real motions. At about this state of the investigation I heard from Professor van Rhijn that he had found on some plates of Præsepe taken with the Radcliffe telescope, which he was reducing, a magnitude equation in declination of exactly the same kind as that shown in Table I. The drifts could not cause any differential proper motions in stars belonging to the cluster, and so this confirmed the fact that the effect was very largely an error.

The only cause which I could think of as capable of producing the effect, was change in the optical field of the object-glass. Optical distortion will not, of course, affect proper motions, unless a change in it, such as could be produced by change in the relative positions of the two components of the object-glass, occurs between the two epochs at which the plates are taken. That maladjustment of the lenses is capable of producing magnitude equation was pointed out by Turner some years ago.* Now the two component lenses were separated for cleaning in 1906 October, 1909 January, and 1914 March, and were readjusted for relative tilt one to the other by the usual method of taking photographs of a bright star with the plate a few millimetres on each side of the focus. Study of these plates failed to reveal any certain departure from symmetry in the nature of these out-of-focus images after the final adjustment on each occasion. So that within the sensitiveness of this method of testing, the object-glass had been adjusted after each cleaning to its original state.

It seemed possible, however, that the test employed was not sufficiently sensitive, and that at one or more of the adjustments a change of tilt had occurred, which was sufficient to cause an error of the nature

* *M.N.*, **65**, 54, 228.

found. The adjustment in 1914 occurred between the exposures at the first and second epochs of all the ten Selected Areas considered above, and so the magnitude equation could be ascribed to a change at that date. The first epoch exposures on the Præsepe plates were made, some before the adjustment in 1906, and some between those in 1909 and 1914, while the second epoch exposures were all made after that in 1914. It was therefore assumed as a working hypothesis that a change in the adjustment of the object-glass took place in 1914, and possibly also in 1906 and 1909, but that between these adjustments the object-glass remained in a constant state.

As the telescope is one that can be used on either side of the pier (all the plates so far considered had been taken with it on the west of the pier), the object-glass can be reversed with respect to a field of stars. It was thus an easy matter to test its present condition, and two plates, one taken through the glass, were exposed on a field, one with the telescope to the west and the other with it to the east of the pier. The field taken was not a very good one for the purpose, as it did not contain many bright stars, but measurement was made of 21 stars of mean magnitude 10·5 and of 20 of mean magnitude 14·5, which showed that the images of the bright stars were shifted relatively to those of the faint ones, when the telescope was on the west of the pier, by + 0″·22 in R.A. and − 0″·15 in Dec. This indicates that there is an error in the present adjustment of the object-glass to which most, if not all, of the magnitude equation found in the proper motions in the ten Selected Areas can be ascribed, the mean interval in epoch between the plates on those areas being eleven years.

Search through the early plates revealed one taken of Præsepe in 1905 February with three exposures, one made with the telescope on the east side of the pier and two with it on the west side. It was presumably taken with the object of testing whether there was any optical distortion, but it had never been measured. A careful measurement has now been made of 144 stars on this plate, and the result shows that at that time there was extremely little magnitude equation. The displacements found are given in Table II.; each group contains twelve stars, and the displacements are referred to 67 comparison stars of mean magnitude 11·8.

TABLE II.

Magnitude Equation existing in 1905, *with Telescope to West of Pier.*

Mag.	Displacement. (unit 0″·001). R.A.	Dec.	Mag.	Displacement. (unit 0″·001). R.A.	Dec.
8·0	+12	+17	11·9	− 4	− 1
9·6	+ 1	+ 6	12·3	+ 1	− 1
10·2	+ 1	+ 4	12·5	0	− 1
10·8	+ 1	+ 3	12·8	+ 4	− 4
11·2	+ 3	+ 1	13·1	+ 3	− 3
11·6	0	+ 1	13·4	− 3	0

Between the date of this plate and those of the first-epoch exposures on the Selected Areas there were two adjustments of the object-glass, those of 1906 and 1909, so that it provides no proof of good adjustment during the latter period. Unfortunately there is no plate which will inform us so easily of the state of the object-glass at that time, but I have hopes of being able to obtain some information by comparing several plates taken of the North Polar region during the period with the mean of the four Greenwich astrographic plates of this region, which should be free from any similar error which may have affected any single Greenwich plate.

It has been already stated that the plates of all the Selected Areas so far considered were taken with the telescope on the west side of the pier, and that the adjustment in 1914 occurred between the taking of the first and second epoch plates. This is not, however, the case with all the areas in our programme. The most northerly of them have in general been photographed with the telescope on the east of the pier, and the results derived from seven of these areas confirm those of Table I., the magnitude equation appearing with the opposite sign. Each area is being photographed in duplicate ; in the first series the exposures at each epoch are on the same plate, which has been stored undeveloped after the first exposure ; in the second series they are on separate plates, that at the second epoch being taken through the glass. In two areas, Nos. 1 and 14, the exposures of one series are separated as before by the adjustment in 1914, but those of the other series were both made after the adjustment. The latter should be free from magnitude equation on the assumption that it is due to a change made in 1914, and consequently the difference of the proper motions found from the two series should give a determination of the magnitude equation. Also in the case of Area No. 16 the exposures of one series were made with the telescope on the west of the pier, while those of the other series were made with it on the east of the pier. On the assumption made as to its cause, the magnitude equation should appear with opposite signs in the two series, and half the difference of the proper motions given by them again provides a determination of the magnitude equation. The results (*b*), (*c*), and (*d*) from these three areas, reduced from proper motions to displacements on the plate by multiplying by the intervals between the epochs, are given in Table III., together with the results (*a*) of the ten areas cited in Table I., similarly reduced to displacements, and those (*e*) of the plates of the field already mentioned as having been taken recently with the telescope on each side of the pier. All five results are seen to be in very fair agreement, especially when we take into account the fact that in the case of (*b*), (*c*), and (*d*) those for the brighter magnitudes depend on a very few stars.

The agreement of the figures under (*e*) with those in the other columns provides reason for believing that any magnitude equation that existed in the period just prior to the adjustment in 1914 was not large, and also that since 1914 there has not been any serious change in the object-glass. The question whether the assumption that the object-glass does not change sensibly except when it is taken apart for cleaning. and whether the effect is the same for different parts of

TABLE III.

Various Determinations of Magnitude Equation, with Telescope on the West of Pier.

Displacements on the Plate (unit 0″·01).

Int. Mag.	R.A.					Dec.				
	(*a*).	(*b*).	(*c*).	(*d*).	(*e*).	(*a*).	(*b*).	(*c*).	(*d*).	(*e*).
B> 9·5	+41	..	..	..	..	−28	..	..	..	..
9·5	+18	..	..	+23	..	−27	..	..	−45	
10·0	+11	+ 2	..	..	..	−17	−42			
10·5	+14	..	..	..	+22	−15	..	..	..	−15
11·0	+ 3	..	+19	+13	..	−20	..	−39	−23	
11·5	+ 3	..	..	..	..	−10				
12·0	+ 2	+21	− 5	+ 3	..	− 4	−32	−12	−12	
12·5	0	..	..	..	..	− 3				
13·0	0	+ 3	− 1	− 1	..	0	− 9	− 5	0	
13·5	0	..	..	..	..	+ 2				
14·0	− 1	+ 4	0	0	..	+ 1	+ 3	+ 1	+ 4	
14·5	− 1	..	..	− 3	0	+ 4	..	..	+ 4	0
15·0	− 1	− 4	− 2	..	..	+ 1	+ 4	+ 6		

(*a*) From P.M.'s of first ten areas reduced.
(*b*) From Area 1.
(*c*) From Area 14.
(*d*) From Area 16.
(*e*) From test plate on east and west of pier.

the sky, as flexure might cause it not to be, will be tested by taking a series of plates of selected fields on each side of the pier. The extremely bad weather of the last few months has as yet prevented the commencement of this work.

It is clear that the method employed for testing the adjustment of the object-glass has not been sufficiently sensitive in the case of our instrument. Is it so in general? The same method is in use at the Royal Observatory, Greenwich, and I have had the privilege of examining similar test plates made there with the 26-inch refractor, on which the definition in the out-of-focus images is on the whole better than it is with the Radcliffe instrument. They also test the adjustment for centring of their lens by means of out-of-focus plates taken through yellow and violet filters. At the suggestion of Mr. C. R. Davidson I have recently taken some similar plates with our instrument, but they also fail to show any certain lack of symmetry in the out-of-focus images. Adjustments of the two lenses of the Greenwich 26-inch refractor are made fairly frequently,* either after cleaning or because test plates show slight change in the adjustment. The accuracy of the determinations of parallax made with that instrument is sufficient proof that the method of testing the adjustment is adequate for that purpose, but the conditions are not in one way so exacting as in the case of proper motion

* See *Observations of Stellar Parallax*, 1913–1924, p. iii.

work, owing to the practice in parallax work of equalising the magnitudes of the stars measured.

In any case the study of out-of-focus images has failed to reveal in the Radcliffe instrument a maladjustment that gives rise to serious magnitude equation, which suggests that photographic refractors that can be reversed on the sky should also be tested in this way. If the telescope cannot be reversed, its optical field can be tested by taking a plate of some standard field, in which the positions are known with great accuracy, but this is a less easy method, as it involves absolute as opposed to differential measurement. The figures given in Table II., especially those for declination, show how sensitive a test can be made by reversing the object-glass on the sky, the unit being equivalent to 1/30,000 mm. on the plate. It has, however, the disadvantage of being rather laborious.

The assumption so far made that there is no sensible change in the object-glass, except when it is taken apart for cleaning, is not borne out by the experience of the Greenwich instrument. There is, however, a difference between the instruments in the way their cells are connected. In the early years the two cells of the Radcliffe object glass were kept apart by wedges and adjusting screws, as are those at Greenwich, the gap being covered in a way that let in moisture to an inconvenient extent. This was changed at the adjustment of 1909, and the two cells were screwed rigidly together, connected by a solid iron ring, the adjustment for tilt of one lens to the other being made by the insertion of strips of paper under the crown lens in its cell. It seems unlikely, therefore, that any appreciable change can take place except when the lenses are separated for cleaning, which is now only seldom necessary. The inner surfaces have not been cleaned for thirteen years, and yet are cleaner than were those of the Greenwich instrument when I saw them recently, though, as I understood, they had been cleaned within the previous two or three years. It is possible, however, that this advantage of a rigid connection between the cells may be offset by a greater possibility of the introduction of strain, which rather than simple tilt may be the cause of our trouble, though one would expect this to have a greater effect on the out-of-focus images than a small tilt.

It is of the highest importance in proper motion work, especially when the interval between the epochs of observation is short, that the optical field of the object-glass should remain unchanged. Unless, therefore, the Radcliffe instrument is unique—and I can see no reason why it should be—it is clearly desirable, when engaged on such a programme, to make adjustments of the object-glass only when absolutely necessary, and when it is done, to take all possible precautions to see that a magnitude equation is not being introduced. My intention is to leave the object-glass in its present state until our observational programme is completed, and to test the optical field periodically to see whether the magnitude equation remains constant. When a number of the Selected Areas, representative of the whole northern hemisphere, have been reduced, some of which will be affected by the change in 1914 and some will not, it should be possible to derive from their results, and from those of the test plates, sufficiently accurate values of the

magnitude equation to be used for correcting the results of those areas affected by it, provided that the assumption so far made, that no sensible change took place except at the adjustment in 1914, is not contradicted by future data.

Summary.

(*a*) Analysis of the results of the first ten completed areas of the proper motion programme revealed a systematic magnitude equation in both R.A. and Declination, which is too large to be a real motion of the stars.

(*b*) This has been traced to a change in the optical field of the object-glass, made in 1914 when adjusting its component lenses after cleaning, although the usual tests on out-of-focus images showed no change.

(*c*) Attention is drawn to the danger of adjusting an object-glass in the middle of a proper motion programme, unless very sensitive tests are employed to ensure that there has been no change in the optical field. It is suggested that the usual method of testing by means of out-of-focus images is not sufficiently sensitive.

Radcliffe Observatory, Oxford :
1927 March 3.

The Errors of Photographic Proper Motions.

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

Summary.

1. A pair of plates of 6 Cygni, one taken directly and the other taken through the glass, with only a few days' interval between them, have been measured and reduced precisely as if they had been proper-motion plates. The residuals in the usual linear equations are consequently free from proper motion, and are merely the results of accidental and systematic errors.

2. As regards accidental errors, a residual is due to (*a*) the error of measurement of the displacement between corresponding images, and (*b*) errors in the computed plate constants.

3. As regards (*b*), it is shown that when the plate constants are calculated from the measures made on 24 comparison stars, the extremely simple method of Christie and Dyson is very slightly inferior to the laborious least-squares solution sometimes adopted, and further, that when the larger values of the errors (*a*) are considered in combination with (*b*), this slight inferiority practically disappears.

4. The measures have been subjected to several tests. In terms of proper motion with an assumed interval of 17·6 years, the probable error of a single determination of proper motion is calculated to be $\pm$ 1″·0 per century in each co-ordinate and for six separate comparisons the probable error is $\pm$0″·4 per century.

5. Magnitude error is not evident from the measures in declination, and its possible existence in right ascension has not been consistently demonstrated.

§ 1. The importance of obtaining reliable indications of the precision with which photographic proper motions are derived need hardly be emphasised. For example, in dealing with the systematic motions of faint stars, the probable errors of the proper motions must be reasonably small if investigations of this kind are to have any definite value. In a recent paper,* I have attempted to obtain an estimate of the probable errors of the proper motions in three of the Cambridge photographic regions; this estimate was based on the internal discordances of the several measures made for each star. The conclusion reached was that, for an average of six comparisons of pairs of images, the mean probable error was of the order of $\pm 0''{\cdot}4$ per century in each co-ordinate. In this discussion, the possibility of the existence of a systematic error such as "magnitude error" was not taken into account. The reduction of the relative proper motions to absolute proper motions, by way of the mean parallactic motions of groups of stars of magnitudes ranging approximately from 8 to 10 on the Harvard Scale, provides a certain amount of material for assessing the possible values of magnitude error within the limited range of magnitudes mentioned. Generally, it was found, in these reductions, that magnitude error, if it existed, was not conspicuously evident.

§ 2. The relative proper motions p—corresponding to an interval of T years—are derived from linear equations of the type

$$ax + by + c + \mathrm{M} = p,$$

in which M is the measure, in one co-ordinate, between corresponding images of an "early" plate and one taken through the glass. Assuming the plate constants a, b, and c, and the co-ordinates (x, y) of a star all known, the proper motion p is strictly the residual in the above equation. The residual comprises the unknown proper motion of the star and all the errors, accidental and systematic, resulting from the measures M and the computed values of a, b, and c. If there are six separate comparisons for each star, the mean of the six residuals is taken to be the proper motion of the star, and the probable error, referred to above, is computed on this basis. If the value of the mean of these six values of the residuals represented really and truly the star's relative proper motion, the probable errors derived as in the paper quoted would, of course, be the true probable errors. In actual practice, this assumption is taken for granted, but it need not, nevertheless, be more than approximately true.

§ 3. This note deals with the consideration of the accidental and systematic errors of photographic proper motions, the proper motions themselves being definitely known so that the residuals in the linear equations are simply the results of errors. The field of 6 Cygni was put on the proper-motion programme in 1926 September and the necessary number of "reversed" plates was secured. About the same time,

* *M.N.*, **87**, 126.

the field was photographed directly. A pair of plates—No. 1716 of 1926 September 12 and No. 1733 of 1926 September 30—the former photographed through the glass and the latter photographed directly—were measured in the usual way by Mr. Green and the writer. The proper motions are therefore all zero and the residuals in the linear equations are consequently due entirely to accidental and systematic errors. The " reversed " plate contains two exposures, denoted by R and S, of 30 minutes each ; the direct plate contains four exposures, denoted by A, B, C, and D, of 15, 9, 5, 3 minutes respectively. It was anticipated that the choice of exposure times would result in the images R and S being of nearly the same size and density as the images B ; however, it was discovered subsequently that the object-glass had been slightly dewed during the exposures on September 30 and consequently the images A to D are very much below the standard—in size and density—expected of the exposures given. On the " reversed " plates there are measurable images of 120 stars ; on the " direct " plates there are measurable images of only 56 stars for all exposures, 14 for the three exposures A to C and 28 for the two exposures A and B. The images A and R are nearly alike. The test to which the measures have been applied is thus a more searching one than was originally intended.

In the ordinary routine work of deriving proper motions, a proposal to compare the images R and S with the series of diverse images A to D would be vetoed owing to fear of magnitude error, and so in practice it is arranged—by choosing suitable exposures—that the corresponding images on the " direct " and " reversed " plates are as nearly alike in quality as possible. In this investigation, the diversity in the series of images A to D was expressly intentioned in the expectation that some light might be thrown on the existence and amount of magnitude error. Accordingly, the components of the displacements between the images R and the images A and B were measured ; similarly, for the images S and the images C and D.

§ 4. It is sometimes the custom to derive the plate constants by a least-squares solution of the N equations,

$$ax + by + c + \mathrm{M} = 0$$

corresponding to N selected comparison stars. It is not generally realised that the simple and much less laborious method suggested by Christie and Dyson * over thirty years ago gives practically as good a determination of the constants as the more ponderous solution by least squares, as was pointed out by Hinks and Russell † in their discussion on stellar parallaxes. The Cambridge proper motions have all been reduced by this simple method with little loss of accuracy (as direct comparisons with the least-squares method, in specific instances, have always shown) and a great economy in computing. Whatever method is employed, the accidental error in the deduced proper motion of any star in the field is made up of the accidental error in the measured quantity M and the error resulting from the slightly inaccurate computed

* *M.N.*, **55**, 61.

† *M.N.*, **65**, 781. (See also H. N. Russell, *Determination of Stellar Parallax*, Carnegie Institution of Washington, 1911, p. 21.)

values of a, b, and c. This last error can be compared in the case of the two methods ; for this purpose, it is necessary to derive certain formulæ.

§ 5. The method of Christie and Dyson is as follows : n comparison stars are selected in each quarter of the plate so that the centres—G_1, G_2, G_3, G_4—of position of the groups are as nearly symmetrically disposed about the centre as possible. We shall assume that the four centres G have the co-ordinates ($\pm$ X, $\pm$ Y) ; generally, X and Y will have the same numerical values. For proper motion work the number n has generally been between 6 and 10, so that the solution of the equations depends on at least 24 comparison stars. Adding all the n equations corresponding to the comparison stars in quarter I., and similarly for the other quarters, we reduce the $4n$ equations of condition to the following four :—

$$n(-aX + bY + c) + M_1 = 0 \quad . \quad . \quad . \quad (1)$$

$$n(+aX + bY + c) + M_2 = 0 \quad . \quad . \quad . \quad (2)$$

$$n(-aX - bY + c) + M_3 = 0 \quad . \quad . \quad . \quad (3)$$

$$n(+aX - bY + c) + M_4 = 0 \quad . \quad . \quad . \quad (4)$$

in which M_1 denotes the sum of the M's for the n stars in quarter I. etc. (2) and (4) give, by addition,

$$2n(aX + c) + M_2 + M_4 = 0 \quad . \quad . \quad (5)$$

and (1) and (3)

$$2n(-aX + c) + M_1 + M_3 = 0 \quad . \quad . \quad (6)$$

Similarly, we have from (1) and (2) and from (3) and (4),

$$2n(bY + c) + M_1 + M_2 = 0 \quad . \quad . \quad (7)$$

$$2n(-bY + c) + M_3 + M_4 = 0 \quad . \quad . \quad (8)$$

Eliminating c between (5) and (6), we have

$$a = \frac{1}{4nX}(M_1 - M_2 + M_3 - M_4) \quad . \quad . \quad (9)$$

and similarly

$$b = \frac{1}{4nY}(M_3 + M_4 - M_1 - M_2) \quad . \quad . \quad (10)$$

The four equations (5) to (8) are clearly not independent, for the addition of (5) and (6) and the addition of (7) and (8) lead to the same value of c, namely,

$$c = -\frac{1}{4n}(M_1 + M_2 + M_3 + M_4) \quad . \quad . \quad (11)$$

Let r denote the probable error of a measure M; r_a, r_b, and r_c the probable errors of the plate constants a, b, and c. Then from (9), (10), and (11) we have

$$r_a = \frac{r}{X\sqrt{N}}; \qquad r_b = \frac{r}{Y\sqrt{N}}; \qquad r_c = \frac{r}{\sqrt{N}},$$

where $N \equiv 4n$ is the total number of comparison stars.

For any star with co-ordinates (x, y) on the plate, the probable value ρ of a residual (or the probable error of the quantity $ax + by + c$)—due to errors in the computed values of a, b, and c—is *apparently* given by

$$\rho = \{x^2 r_a^2 + y^2 r_b^2 + r_c^2\}^{\frac{1}{2}}$$

or

$$\rho = \frac{r}{\sqrt{\mathrm{N}}}\left\{1 + \frac{x^2}{\mathrm{X}^2} + \frac{y^2}{\mathrm{Y}^2}\right\}^{\frac{1}{2}} \quad . \quad . \quad . \quad (12)$$

but Professor Eddington has pointed out to me that, although the last formula (12) is correct, the conditions of symmetry being assumed, *i.e.* X = Y, the method of deduction is erroneous owing primarily to the non-independence of the four equations (5) to (8). Actually, we have, writing X for Y,

$$ax + by + c = \frac{1}{4n\mathrm{X}}\{(x - y - \mathrm{X})\mathrm{M}_1 + (-x - y - \mathrm{X})\mathrm{M}_2 + (x + y - \mathrm{X})\mathrm{M}_3 + (-x + y - \mathrm{X})\mathrm{M}_4\}$$

$\mathrm{M}_1 \ldots \mathrm{M}_4$ are each the sum of n quantities M, and therefore the probable error of $\mathrm{M}_1 \ldots \mathrm{M}_4$ is each $r\sqrt{n}$ numerically. Hence the probable error ρ of the expression $(ax + by + c)$ is given by

$$\rho = \frac{r\sqrt{n}}{4n\mathrm{X}}\{(x - y - \mathrm{X})^2 + (-x - y - \mathrm{X})^2 + (x + y - \mathrm{X})^2 + (-x + y - \mathrm{X})^2\}^{\frac{1}{2}},$$

which reduces to

$$\rho = \frac{r}{\sqrt{\mathrm{N}}}\left\{1 + \frac{x^2 + y^2}{\mathrm{X}^2}\right\}^{\frac{1}{2}}$$

equivalent to (12) when Y = X.

If the comparison stars are evenly distributed over the plate, the co-ordinates of the corners of the plate will be $(\pm 2\mathrm{X}, \pm 2\mathrm{Y})$. Hence the probable value of a residual for a star in an extreme corner of the plate will be $\frac{3r}{\sqrt{\mathrm{N}}}$ numerically. For a star at the centre, it will be $\frac{r}{\sqrt{\mathrm{N}}}$.

To obtain some idea of what the probable error is on the average, let us suppose that the number of stars on the plate is very large. We still assume that the comparison stars are evenly distributed. If ϵ denotes the mean square value of the probable errors of all the stars, we have

$$\epsilon^2 \int_{-2\mathrm{X}}^{+2\mathrm{X}} \int_{-2\mathrm{Y}}^{+2\mathrm{Y}} dxdy = \frac{r^2}{\mathrm{N}} \int_{-2\mathrm{X}}^{+2\mathrm{X}} \int_{-2\mathrm{Y}}^{+2\mathrm{Y}} \left(1 + \frac{x^2}{\mathrm{X}^2} + \frac{y^2}{\mathrm{Y}^2}\right) dxdy,$$

or, with an obvious change of variables,

$$\epsilon^2 = \frac{r^2}{4\mathrm{N}} \int_0^2 \int_0^2 (1 + u^2 + v^2) dudv$$

from which

$$\epsilon = \frac{1 \cdot 9r}{\sqrt{\mathrm{N}}}.$$

If we neglect stars outside the circle of radius $2X(\equiv 2Y)$ we obtain a simple expression for the value $\bar{r}$ of the average probable error; it is given by

$$\bar{r}\int_0^{2X}\int_0^{2\pi} R dR d\theta = \frac{r}{\sqrt{N}}\int_0^{2X}\int_0^{2\pi}\left(1+\frac{R^2}{X^2}\right)R dR d\theta,$$

for which

$$\begin{aligned}\bar{r} &= \frac{r}{4\sqrt{N}}\int_0^4 (1+w)^{\frac{1}{2}}dw \\ &= \frac{r}{4\sqrt{N}}\{(5)^{\frac{3}{2}}-1\} \\ &= \frac{1{\cdot}7r}{\sqrt{N}}.\end{aligned}$$

For the actual distribution of a finite number of stars, we shall not err badly if we assume that the average or the mean square value of the residuals is $\frac{2r}{\sqrt{N}}$.

In these formulæ, we still require the value of r, the probable error of the measures M of the n comparison stars. From the measures of the pair of plates of 6 Cygni, these of course can be found in terms of seconds of arc. But to express them in terms of proper motion, I am going to assume that the interval between the plates is 17·6 years, so that one unit of M on the measuring machine corresponds to 0″·1 per century in terms of proper motion. This assumed interval is 3 or 4 years less than the average interval on which the Cambridge proper motions are based—it has been chosen, however, merely to simplify the computations.

From the 24 comparison stars, the probable error of a single measure of M is—

In right ascension, ± 0″·96 per century.
In declination ± 0″·81 „ „

We shall take the numerical value of r to be 1″ per century, in round figures.

Using this value of r, we find that the probable value of a residual for stars near the centre of the plate is ± 0″·2 per century, for stars in the extreme corners ± 0″·6 per century, and as an average of all the stars, ±0″·4 per century. It is to be remembered that the formulæ and numerical results so far obtained deal with the effect of the errors in the *values of* M *of the comparison stars only*, that is to say, with the effect of the errors in the computed values of the plate constants.

§ 6. Let us now compare the results which we should have obtained from a least-squares solution of the N equations

$$ax + by + c + M = 0.$$

This will be equivalent to obtaining corrections Δa, Δb, and Δc to the values of the constants obtained, from the Christie and Dyson method, from the equations

$$x\Delta a + y\Delta b + \Delta c = v,$$

v being the residual as before. Forming the normal equations in the usual way, we have

$$\begin{aligned} \Delta a[xx] + \Delta b[xy] + \Delta c[x] &= [xv] \\ \Delta a[xy] + \Delta b[yy] + \Delta c[y] &= [yv] \\ \Delta a[x] + \Delta b[y] + \Delta c\,\mathrm{N} &= [v] \end{aligned}$$

But from the supposed uniform distribution of the comparison stars, we shall have, theoretically,

$$[xy] = [x] = [y] = 0.$$

In practice, these quantities will be so small as to be wholly negligible in comparison with $[xx]$, $[yy]$, and N. Accordingly,

$$\Delta a = \frac{[xv]}{[xx]}; \qquad \Delta b = \frac{[yv]}{[yy]}; \qquad \Delta c = \frac{[v]}{\mathrm{N}}.$$

The probable value of the residuals v is still r. The probable errors of a, b, and c—denoted by r_a', r_b' and r_c'—from the least-square solution are then

$$r_a' = \frac{r}{\sqrt{[xx]}}; \qquad r_b' = \frac{r}{\sqrt{[yy]}}; \qquad r_c' = \frac{r}{\sqrt{\mathrm{N}}}.$$

The two methods give the same probable error in the computed values of c. The comparison of the two methods depends, then, on the values of the quantities $[xx]$ and $[yy]$.

In deriving the plate constants for the reduction of the relative proper motions of 250 stars around Σ443, Rambaut in 1913 used the least-squares method with 200 stars as comparison stars. This number is so large that we can easily obtain theoretical values of r_a', etc. Let μdx be the number of comparison stars between the abscissæ x and $x + dx$. Then if N is the total number of comparison stars,

$$\mathrm{N} = 2\mu\int_0^{2\mathrm{X}} dx = 4\mu\mathrm{X},$$

and

$$[xx] = 2\mu\int_0^{2\mathrm{X}} x^2 dx = \tfrac{4}{3}\mathrm{N}\mathrm{X}^2.$$

Hence,

$$r_a' = \frac{0{\cdot}87r}{\mathrm{X}\sqrt{\mathrm{N}}}, \quad \text{etc.},$$

and for a star with co-ordinates (x, y), the probable value ρ' of a residual due to the errors in the measures M of the comparison stars, is given by

$$\rho' = \frac{0{\cdot}87r}{\sqrt{\mathrm{N}}}\left\{1 + \frac{x^2}{\mathrm{X}^2} + \frac{y^2}{\mathrm{Y}^2}\right\}^{\frac{1}{2}}.$$

The simple method would have given

$$\rho = \frac{r}{\sqrt{\mathrm{N}}}\left\{1 + \frac{x^2}{\mathrm{X}^2} + \frac{y^2}{\mathrm{Y}^2}\right\}^{\frac{1}{2}},$$

showing that the least-squares method has a very trifling gain in precision out of all proportion to the tremendous amount of extra labour expended in attaining it.

Returning now to the 24 comparison stars of the field of 6 Cygni, we can likewise make a simple calculation of the values of $[xx]$ and $[yy]$. Let us assume simply that the x co-ordinates of the six stars in each quarter of the plate have numerical values

$$\frac{X}{3}, \quad \frac{2X}{3}, \quad \ldots, \quad \frac{6X}{3}.$$

We then find that $[xx] = \frac{3}{2}\,NX^2$, and, consequently,

$$\rho' = \frac{0''.81r}{\sqrt{N}}\left\{1 + \frac{x^2}{X^2} + \frac{y^2}{Y^2}\right\}^{\frac{1}{2}}.$$

Comparison with the results obtained from the simple method shows that, in this case, the gain in accuracy from using the least-squares method is represented by the ratio of 4 to 5 in the probable errors obtained by the two methods Using our values of r and N ($r = 1''.0$ per century, $N = 24$) we find that by least squares a gain of $0''.1$ per century at most in the deduced values of the probable errors for stars at the corners of the plates, is the result.

§ 7. So far, we have been dealing only with the errors arising from the computed values of the constants a, b, and c. If ρ_M denotes the probable error of a measure M for any star with co-ordinates (x, y), the probable value of a residual in the equation

$$ax + by + c + M = 0$$

is by (12)

$$(\rho^2 + \rho_M{}^2)^{\frac{1}{2}}.$$

Now we have seen that ρ varies between $0''.2$ and $0''.6$ per century when Christie and Dyson's method is employed, and in the least-squares method we may take the corresponding limits to be $0''.1$ and $0''.5$ per century. If we assume that ρ_M is $1''.0$ per century (it will be shown later that the measures of the pair of plates of 6 Cygni indicates this to be of the right order of magnitude), the total probable value of a residual, arising from errors in the plate constants and errors in the measured quantities M, is influenced to a very small extent by the values of ρ or of ρ', so that, practically, it is immaterial whether we use $(\rho^2 + \rho_M{}^2)^{\frac{1}{2}}$ or $(\rho'^2 + \rho_M{}^2)^{\frac{1}{2}}$ in our calculations. It appears, then, that the small gain in precision (as regards the computation of the plate constants) is entirely illusory, as it is swamped by the very much larger errors in the quantities M.

§ 8. There is one point of some importance that has not been stated; it is this. When we seek to derive relative proper motions, we know nothing *a priori* of the individual proper motions of the comparison stars. A preliminary solution, however, enables the larger proper motions to be detected, and so it is generally possible to exclude from the final selection of comparison stars all those with relative proper motions exceeding, let us say, $2''$ per century in either co-ordinate.

The residuals v for the comparison stars consequently include the small relative proper motions of these stars in addition to the accidental errors of the measured quantities M. In the determination of the plate constants—whatever the method of solution—it is virtually assumed that the small proper motions are of the character of accidental errors. Therefore, our value of r is an underestimate. Actually,

$$r^2 = r_M^2 + \rho_c^2,$$

where r_M is the probable error of the measures M for the comparison stars and ρ_c is the probable value of the relative proper motions of these stars. It is impossible, in general, to state definitely what the value of ρ_c is likely to be. Probably it will not exceed $0''{\cdot}8$, so that in this event our value of r requires to be multiplied by a factor 1·3. If this is so, our conclusions in the preceding section still hold good.

§ 9. Before proceeding to the consideration of the actual measures of the two plates of the region round 6 Cygni, we may summarise the preceding theoretical discussion in its application to the determination of relative proper motions, as follows : Regarding all errors as accidental errors, we find that the deduced relative proper motion of any star in a photographic region is subject to error arising from (*a*) the error of measurement of the quantity M, and (*b*) the errors in the computed values of the plate constants *a*, *b*, and *c*. Further, we find that the errors (*a*) are of very much greater importance numerically than the errors (*b*)—the latter can be made as small as we please by increasing as far as possible the number of suitable comparison stars. Finally, we conclude that in view of the numerical values of (*a*), the supposed slight gain in precision following the employment of the least-squares method of solution is, in practice, almost wholly negligible.

§ 10. In all, 98 stars were measured on the combined plates of 6 Cygni. Of these, 56 had all four images of the direct exposures A to D measurable, 14 had A to C measurable, and the remainder, 28 in number, had A and B measurable. The images R were combined, for measurement, with the images A and B, S with the images C and D. The 56 brightest stars have thus four comparisons, 14 have three comparisons, and 28 only two comparisons. The comparisons RA and RB are not, of course, strictly independent—and similarly for SC and SD—but having regard to the diversity of the series of images A to D, this partial non-independence has not been insisted on in certain parts of the discussion following.

The 56 brightest stars were subdivided into four groups according to brightness. Details regarding the 98 stars are summarised in the following table.

The average diameters are in terms of the scale divisions of the measuring machine, and the approximate average magnitudes are on the Harvard Scale.

§ 11. Twenty-four comparison stars were selected in accordance with the principles of Christie and Dyson's method. They belong with a few exceptions to Groups III. and IV. of Table I. The residuals in the linear equations of these stars enable us to compute the value of r defined in § 4. It is to be remembered that, in order to express our

TABLE I.

Group.	Number of Stars.	Average Diameter.	Approximate Average Magnitude.	Number of Comparisons.
I.	11	42	8·2	4
II.	14	28	9·5	4
III.	15	21	10·4	4
IV.	16	17	10·8	4
V.	14	15	11·2	3
VI.	28	13	11·6	2

results in terms of proper motion, we have—for simplicity in the calculations—assumed an interval of 17·6 years between the plates. The probable value r of a single residual is found to be (numerically)—

In right ascension	0″·96 per century.
In declination	0·81 ,, ,,

§ 12. For any star in the field, we have four residuals in each co-ordinate of the stars belonging to Groups I. to IV. of Table I., three residuals in each co-ordinate of the stars belonging to Group V., and two residuals in each co-ordinate of the stars in Group VI. If we had been dealing actually with two proper motion plates separated by a real interval of 17·6 years, we should regard the algebraic means of the residuals as the relative proper motions of the stars in the field. Proceeding in this way with the two plates of 6 Cygni, we obtain for each star a fictitious proper motion in each co-ordinate. These fictitious proper motions ought, of course, theoretically, to be all zero ; the probable values of these quantities (taken without regard to sign) yield then a reliable estimate of the probable errors of true photographic proper motions. In terms of proper motion, the results are in columns 2 and 3 of Table II. below.

TABLE II.

Group.	Probable Error (per century).		Average P.M. (per century).	
	R.A.	Declination.	R.A.	Declination.
I.	±0″·60	±0″·52	+0″·33	+0″·21
II.	0·53	0·44	+0·21	+0·40
III.	0·41	0·36	−0·01	+0·14
IV.	0·51	0·47	−0·20	−0·03
I. to IV.	0·49	0·43	+0·05	+0·16
V.	0·83	0·52	+0·46	−0·25
VI.	0·89	0·72	+0·23	+0·24

For Groups V. and VI. the values in the second and third columns are higher than for Groups I. to IV., as we should expect from the smaller number of comparisons. A noticeable feature is the smaller values of the probable errors in declination as compared with the values in right ascension.

In columns 4 and 5 the algebraic means of the fictitious proper

motions are given : these ought to furnish evidence of magnitude error, if it existed. If we confine our attention, for the moment, to Groups I. to IV., there appears to be very little trace of magnitude error in declination. In right ascension, however, it looks as if there are signs as the average proper motion decreases almost uniformly from + 0″·33 in Group I. to − 0″·20 in Group IV. But there the progression stops, for Groups V. and VI., comprising 42 stars, fail to carry it on. The conclusion from this table seems to be that, in declination, the errors are purely accidental in character, and that, in right ascension, there is some suspicion of systematic error. We shall consider again the question of magnitude error.

§ 13. Let us now consider the individual residuals from each comparison of a pair of stellar images. Each individual residual may be regarded as the error in a single determination of proper motion. For stars in Groups I. to IV., we form the sum of the four residuals (taken without regard to sign) and similarly for Groups V. and VI. The sum total of the residuals for all the stars in a group provides the material for estimating the probable error of a single observation of proper motion. The results are as follows (columns 2 and 3) :—

TABLE III.

Group.	P.E. of a Single Observation (per Century).		P.E. of a Single Observation (Mean Discordances).	
	R.A.	Declination.	R.A.	Declination.
	″	″		
I.	±0·90	±0·69		
II.	0·80	0·65		
III.	0·82	0·77		
IV.	1·14	0·87		
	″	″	″	″
I. to IV.	0·91	0·76	±0·86	±0·68
V.	1·27	0·94	1·21	0·92
VI.	1·10	1·02	0·88	0·86

Again, the results in declination are definitely superior to the results in right ascension. The high values in Group IV. (as compared with Groups I. to III.) are probably due to the rather faint images of the exposure D of several stars in this group : one or two of such stars ought properly to have been included in Group V.

These probable errors include the probable errors in measuring the quantities M (of our formulæ) and the probable errors arising from those in the computed values of the plate constants. We have seen that the latter are, on the average over the plate, ± 0″·4 per century. The probable errors in the above table are therefore mostly due to the accidental errors of measurement of the quantities M. In round figures, we can take, consequently, the probable error to be ± 1″·0 per century for a single observation in each co-ordinate averaged over the 98 stars (this is a slight underestimate in right ascension and a rather greater overestimate in declination).

It is interesting to see the relation between the probable errors of a single observation obtained from the mean discordances from the

fictitious proper motions and the probable errors given in columns 2 and 3 of Table III. The former values are given in columns 4 and 5 of the above table—they correspond to the probable errors which I have derived from the internal discordances of measured proper motions. As might be expected, these values come out rather smaller than the corresponding values of columns 2 and 3. The data above seem to indicate that the probable errors of proper motions deduced on a previous occasion * require to be increased by about 10 or 15 per cent.

§ 14. I have divided the 56 stars with four observations into four groups (*a*), (*b*), (*c*), and (*d*), depending on the distances of the images from the centre of the plate. Theoretically, the probable error of a single observation is $(\rho^2 + \rho^2_M)^{\frac{1}{2}}$ numerically—by the formula of § 7—and as the numerical value of ρ is estimated to increase from 0″·2 to 0″·6 per century from the centre of the plate outwards, the stars in the groups (*a*) to (*d*) ought to show, on the average, a slight increase in the value of the probable error, ρ_M being rather less than 1″·0 per century in right ascension and declination. The calculations in Table IV. below are based on the sums of the individual residuals (taken without regard to sign).

Table IV.

Group.	Number of Stars.	P.E. of a Single Observation.	
		R.A.	Declination.
(*a*)	13	±0″·88	±0″·82
(*b*)	14	0·90	0·67
(*c*)	19	0·93	0·71
(*d*)	10	1·00	0·90

In right ascension there is a slight progression, agreeing pretty well with the formula : in declination, group (*a*) appears to be exceptional.

§ 15. The grouping of the stars according to the quarters (I. to IV.) of the plate in which the images are situated leads to the following analysis of the average fictitious proper motions (the numbers in parentheses are the numbers of the stars to which the average values refer).

Table V.

Group.	R.A.				Declination.			
	i.	ii.	iii.	iv.	i.	ii.	iii.	iv.
I. to IV.	+0″·11	+0″·31	−0″·23	+0″·12	+0″·21	−0″·07	+0″·32	+0″·18
	(10)	(12)	(16)	(18)	(10)	(12)	(16)	(18)
V.	+0·70	+0·27	+0·07	+0·68	+0·70	−1·03	−0·43	−0·8
	(2)	(3)	(3)	(6)	(2)	(3)	(3)	(6)
VI.	+0·70	+0·27	+0·17	−0·16	+0·08	+0·01	+0·70	+0·13
	(6)	(9)	(6)	(7)	(6)	(9)	(6)	(7)
All stars	+0·36	+0·29	−0·10	+0·27	+0·22	−0·16	+0·32	+0·12
	(18)	(24)	(25)	(31)	(18)	(24)	(25)	(31)

* *M.N.*, **87**, 126.

The mean fictitious proper motion of all the stars is + 0″·16 per century in right ascension and + 0″·12 per century in declination. Otherwise, the small average values of Table V. appear to be wholly accidental in character.

§ 16. The final test to which the measures have been applied is the investigation of a possible systematic difference between the means of the comparisons RA and RB and the means of SC and SD. The images R and S are practically identical in quality; the images A to D form a series of diminishing size. The images A and B as a pair are somewhat inferior to R in size: on the other hand, the images C and D are markedly inferior in size to the images S. Stated otherwise, the mean of the images A and B is approximately one and a half magnitudes brighter than the mean of C and D. The following table exhibits the mean fictitious proper motion calculated separately from the comparison of R with A and B and from the comparison of S with C and D, for all the four groups of stars with four observations. The last line of the table gives the probable value of a fictitious proper motion for all the stars. Were magnitude error of any numerical importance, some definite systematic difference between the values under the headings R : AB and S : CD ought to be shown up. In declination, the differences alternate in sign, suggesting that the entries in the fourth and fifth columns are almost certainly accidental in character. In right ascension the differences for the four groups appear to be not wholly systematic in character.

TABLE VI.

Group.	Right Ascension.			Declination.		
	R : AB.	S : CD.	Difference.	R : AB.	S : CD.	Difference.
	″	″	″	″	″	″
I.	+0·50	+0·13	+0·37	+0·28	+0·16	+0·12
II.	+0·48	−0·05	+0·51	+0·36	+0·48	−0·12
III.	−0·27	+0·17	−0·44	+0·48	−0·18	+0·66
IV.	−0·18	−0·21	+0·03	−0·19	+0·12	−0·31
	±0·82	±0·73	..	±0·58	±0·65	

§ 17. *Conclusion.*—The several tests to which the measures have been applied appear to lead to the following conclusions regarding the accuracy of photographic proper motions.

(i.) For an interval of 17·6 years, the probable error of a single determination of proper motion is ± 1″·0 per century. For a minimum of 6 separate comparisons, the probable error is consequently ± 0″·4 per century. This is in almost exact agreement with the results previously found for three regions, the intervals, however, being somewhat longer.

(ii.) Magnitude error is not evident in declination, and if it exists in right ascension the measures are not consistent as to its elucidation. For two proper-motion plates on which corresponding images are of equal quality and size, it seems more than probable that magnitude error is inappreciable in amount.

Observatory, Cambridge:
1927 March 5.

Personality in Meridian Observations of the Sun and the Equinox Correction. By J. Jackson, M.A., D.Sc., and R. T. Cullen.

(*Communicated by the Astronomer-Royal.*)

A good deal of attention has been given recently to a comparison of the observed and computed positions of various bodies of the solar system in order to test the uniformity of the earth's rotation. The most important observations refer to the sun and moon, and as the sun moves comparatively slowly it is most important to remove systematic errors in the observations of it. In a paper * in which the observations of sun made at Greenwich in the period 1851–1924 were discussed by Mr. Cullen, a discontinuity amounting to $0^s.04$ was found in 1915 the date when the impersonal micrometer was introduced. Professor E. W. Brown in *Yale Transactions*, vol. iii., part 6, has not applied the correction found by Mr. Cullen, but his results will obviously be improved by its application. Dr. Innes and Dr. H. Spencer Jones have accepted the application of this correction to reduce the later observations to the same basis as the earlier ones. Dr. Fotheringham, however, has preferred to leave the observations unaltered, as he finds that this gives a closer agreement in the fluctuations of the sun and moon from their computed positions. In view of the importance of the subject, we have decided to make a more detailed examination of the observations.

The following table gives the difference between the observed and computed R.A.'s of the sun for each year from 1901–1925 for each observer, and the weighted mean $\Delta'\alpha$ for each year. The values given in this table after 1914 should be increased by $+^s.005$ for change in adopted equinox. It will be seen at once that after 1914 the different observers agree amongst themselves much better and the variations from one year to the next are smaller. For example, observer B differed systematically from the mean in the years 1901–1905 by at least $-^s.04$, while R.C. showed a large residual in the opposite sense for the years 1909–1913.

The question really to be settled is whether the change of about $^s.04$ which appeared after 1914 was due to change of personality in the observers or to change in the sun's position. It has been suggested (*Observatory*, **50**, 65) that the observations might be made by means of both the fixed wires and the impersonal micrometer, so that the results might be directly compared. This would, of course, involve the observation of clock stars as well as the sun with the fixed wires, and it is doubtful if observations extending over a whole year would be sufficient. The four observers of 1913 and 1914 showed changes of $+^s.01$, $-^s.06$, $+^s.02$, and $+^s.05$ respectively from one year to the next.

To solve the question, the quantity headed $\Delta\lambda$ in the last column of the table is important. There is no reason to suspect that there was any change (of a seasonal nature) which was introduced into the declination observations in 1915. Now when the residuals in declination are analysed in the form $x\cos\alpha + y\sin\alpha$ the coefficient x is $\Delta\lambda\sin\epsilon$,

* *M.N.*, **85**, 1000.

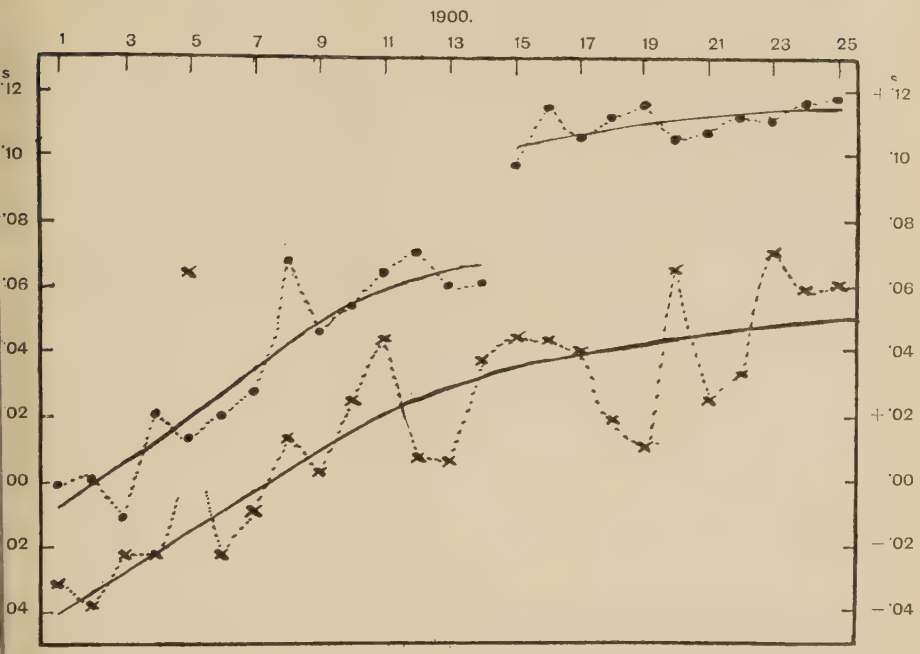
TABLE I.

Year 1900 +.	Observed—Computed R.A. of Sun.												From Decs.	
	J. J.	B.	W.	J. S.	R. C.	E.	B. E.	H. A.	F. J.	H. B.	L. S.	G. R.	$\Delta' a$.	$\Delta\lambda$.
	s	s	s	s	s	s	s	s	s	s	s	s	s	s
1		−·06											−·001	−·031
2		−·04	−·03	·00									+·001	−·038
3		−·05	−·03	−·04	·00								−·011	−·022
4		−·02	·00	−·01	+·04	·00							+·021	−·022
5		−·05	+·04	+·01	+·02	+·03							+·013	+·064
6			+·02	+·05	+·05	·00							+·020	−·022
7			+·01	+·06	−·01	+·04		+·02					+·028	−·009
8			+·02	+·07	+·10	+·05		+·02					+·068	+·013
9			+·01		+·10	+·01	+·02	+·06					+·046	+·003
10			+·06		+·13	+·02	+·01	·00					+·054	+·026
11			+·06		+·16	+·03	+·04	..					+·064	+·043
12			+·04		+·12	+·02	+·04	+·04					+·070	+·008
13			+·03		+·15			+·04	+·09				+·060	+·007
14			+·04		+·09			+·06	+·14				+·061	+·038
15	+·10		+·08		+·06			+·08	..				+·093	+·043
16	+·11		+·10		+·10			+·11	+·14				+·110	+·042
17	+·11		+·11		+·08			+·06	..				+·102	+·040
18	..		+·11		..			+·11	..				+·107	+·019
19	+·10		+·11		+·09			+·11	+·13				+·112	+·010
20	+·09		+·09		+·08			+·10	+·12				+·102	+·066
21	+·12		+·13		+·10			+·09	+·11	+·12			+·103	+·025
22			+·11		+·07			+·13	+·15	+·10	+·12		+·107	+·032
23			+·12		+·10			+·11	+·10	+·09	+·09	+·10	+·106	+·070
24			+·10		+·10			+·10	..	+·08	+·11	+·11	+·113	+·059
25			+·13		+·11			+·11	..	+·11	+·12	+·09	+·114	+·060

1915. April to December only.

where ϵ is the obliquity of the ecliptic and $\Delta\lambda$ is the change in the sun's longitude reckoned from the node—*i.e.* from the equinox determined from the observations of the sun for the year in question. The quantity $\Delta\lambda$ clearly shows no abrupt change in 1915.* The quantity $\Delta' a$ in the last column but one is the correction to the sun's R.A. from the adopted equinox of the clock stars. For the latter the individual positions are derived from Greenwich observations, but they are referred to Newcomb's equinox. The quantities are shown graphically on the diagram and the results from the smooth curves are given in the following table.

* The result for 1905 is discordant. The reason is due to large residuals in the important months near the equinoxes.



Upper broken curve $\Delta'\alpha$.
Lower curve $\Delta\lambda$.

$+^{s}\cdot005$ has been applied to R.A.'s 1915–1921 to reduce to Newcomb's equinox.

TABLE II.

Year.	$\Delta\lambda$.	$\Delta'\alpha$.	Correction to Newcomb's Equinox.	Year.	$\Delta\lambda$.	$\Delta'\alpha$.	Correction to Newcomb's Equinox.
	s	s	s		s	s	s
1901	−·040	−·007	−·033	1915	+·035	+·103	−·068
02	·035	−·001	·034	16	·037	·105	·068
03	·028	+·006	·034	17	·039	·107	·068
04	·022	·012	·034	18	·040	·108	·068
05	·015	·020	·035	19	·041	·109	·068
06	·009	·027	·036	20	·043	·110	·067
07	−·003	·033	·036	21	·045	·111	·066
08	+·004	·041	·037	22	·047	·112	·065
09	·010	·048	·038	23	·048	·114	·066
10	·015	·054	·039	24	·049	·115	·066
11	·021	·059	·038	1925	+·050	+·115	−·065
12	·025	·062	·037				
13	·029	·065	·036				
1914	+·032	+·066	−·034				

The quantities $\Delta\lambda$ show that in the twenty-four years 1901–1925 the sun advanced in longitude (measured from the true equinox) by $^{s}\cdot090$, or $1''\cdot35$ over the tabular value, the advance being more rapid during the first half of the period. The observations in right ascension show a similar gradual change apart from an abrupt jump in 1915. The correction to Newcomb's equinox for 1901–1914 is $-^{s}\cdot036$, for 1915–1925 $-^{s}\cdot067$ if we adopt the observations uncorrected. Everything runs smoothly if we adopt a change of personality in the observations of the sun in R.A. of amount $^{s}\cdot031$ or $''\cdot46$ on the introduction of the impersonal micrometer. We would prefer this change instead of $^{s}\cdot04$ or $''\cdot60$ previously given by Mr. Cullen, which did not take account of the gradual change in $\Delta'\alpha$ in the neighbourhood of 1915.

Dr. Fotheringham has suggested that the abrupt change revealed above was caused to a large extent by change of observers or a sudden change in the personality of one or two observers. There were, in fact, considerable divergencies between the results of different observers before 1915. The observations of W. and H. A., for example, show on the whole results similar to the mean, but R. C. showed a maximum in 1911 higher than anything observed since the introduction of the impersonal micrometer. The best Dr. Fotheringham can do with the observed right ascensions is to bring the broken curve for $\Delta'\alpha$ into a continuous curve by increasing the values for 1913 1914 by $^{s}\cdot02$ or $^{s}\cdot03$ and perhaps slightly lowering the points of the next year or two. This would involve a steady equinox for 1901–1910, then a rapid change round about 1915, followed by a steady equinox again. Or, to express the result geometrically, the observations indicate the following departures from theory :—

(1) A regular advance of the sun's longitude in its orbit from 1901–1925, amounting to $1''\cdot35$;

and if we do not allow for a change of $^{s}\cdot03$ in personality in the observed right ascensions of the sun in 1915—

(2) a rotation of the plane of the ecliptic about the pole of the equator which took place rather rapidly about 1915, altering the equinox by $^{s}\cdot03$ but not altering the position of the sun relative to the equinox.

It seems much simpler to assume a change in the mean personality of the observers in 1915 and then the observed right ascensions and declinations are explained by a gradual advance of the sun in its orbit without alteration of the position of the orbital plane.

The Longitude of the Radcliffe Observatory. By H. Knox-Shaw, M.A., D.Sc., and H. G. Scott Barrett.

The hitherto adopted longitude of the Radcliffe Observatory, $5^{m}\ 2^{s}\cdot6$ west of Greenwich, depends on the transport of chronometers in 1842 by the Rev. Richard Sheepshanks. In the introduction to *Radcliffe Observations*, vol. ii., Manuel Johnson describes this value as

being the result of a most careful chronometrical determination, and expresses the hope that the details will soon be published. This was never done ; only some manuscript notes on the work have been found here. As it seemed likely that this value was in error by a quantity of the order of half a second of time, advantage was taken of the special time-signals broadcast in October, to make a new determination of the longitude.

The transit circle of this observatory is not fitted with a moving-wire micrometer, nor can it be reversed. In view of this and of the great labour involved in trying to push the result to another figure, accuracy to $0^s{\cdot}1$ only was aimed at. This is sufficient for all ordinary purposes. In order to eliminate as far as possible the effects of personal equation, the following method of observing the W/T signals was employed. Every fifth signal in the first, third, and fifth minutes of the rhythmic series sent out at $22\frac{1}{2}^h$ G.M.T. from Paris was tapped on to the chronograph by the same observer as took the transits. The personal equation involved is then the difference of those in observing an expected event aurally and visually. This is not necessarily zero, but experience at Helwan has shown it in general to be small. The observer throughout was HB, but to provide some check on his personal equation, observations were also made on the last night by KS, whose personal equations in taking transits by the tapping method as compared with the moving wire, and of receiving W/T signals by the tapping method, had been determined at Helwan and found to be small,* the former being $+0^s{\cdot}02$ (observation early) and the latter $+ 0^s{\cdot}04$, the difference, $0^s{\cdot}02$, being negligible for our present purpose. The comparison made on 1926 October 23 showed a difference of only $0^s{\cdot}02$ between the two observers, both in taking the transits and receiving the time signals.

Date.	Obs.	Observed Sidereal Time of Signals at Greenwich.	Observed Sidereal Time of Signals at Oxford.	Longitude of Radcliffe Observatory.
		h m s	m s	m s
1926 Oct. 5	HB	23 28 51·02	23 47·99	5 3·03
6	,,	32 47·57	27 44·57	3·00
7	,,	36 44·14	31 41·16	2·98
9	,,	44 37·20	39 34·24	2·96
11	,,	52 30·23	47 27·23	3·00
18	,,	0 20 6·13	15 3·07	3·06
23	,,	39 48·97	34 45·96	3·01
23	KS	39 48·97	34 45·92	3·05
			Mean	5 3·01

Observations were made on seven nights, the transits being taken as near the times of reception of the W/T signals as the weather permitted. The clock stars all lay between $+ 30^\circ$ and $- 10^\circ$ in declination, and the average number observed each night was eight. Their positions were taken from the American Ephemeris and Eichelberger's corrections were applied. The collimation error was determined by collimators,

* *Helwan Observatory Bulletin*, 28.

the level error by the mercury trough, and the azimuth error both by close circum-polars and from solutions of all the stars observed, which ranged in N.P.D. from 120° to 40° S.P. The times of the W/T signals as observed at Greenwich were kindly communicated by the Astronomer Royal. The results are given in the table, the observed times being the means of the beginnings of the first and last signals.

We therefore adopt

Longitude of Radcliffe Observatory (Transit Circle) = $5^m\ 3^s{\cdot}0$ W.

This value is in good agreement with that adopted for the University Observatory at Oxford, which depends on the Ordnance Survey.

Radcliffe Observatory, Oxford :
1927 *March* 10.

Preliminary Values of the Variation of Latitude at Greenwich in 1926.

(*Communicated by the Astronomer Royal.*)

The present paper contains the preliminary values of the latitude variation derived from observations made with the Cookson Floating Zenith Telescope during 1926.

The reductions have been carried out in the same manner as in previous years.

The following table gives the values of the latitude variation, in the sense mean latitude *minus* observed, as derived from the observations :—

Date.	Variation.	Number of Plates.
	″	
1926·026	+·066	27
·176	−·045	21
·276	+·017	19½
·355	+·095	11
·438	+·042	15½
·496	+·056	26
·566	+·050	17
·669	−·022	25½
·752	−·026	16
·845	+·047	5½
·912	−·022	15
1927·050	−·060	26½

From a smooth curve drawn to represent the observed results as closely as possible, the following corrections, applicable to observed north polar distances, were derived :—

1926·0	+″·06	1926·6	+″·02
26·1	+·01	26·7	·00
26·2	−·02	26·8	−·02
26·3	+·04	26·9	−·04
26·4	+·07	1927·0	−·06
26·5	+·05		

The final reduction of the observations from 1919 to 1926 is in hand.

Royal Observatory, Greenwich :
March 1927.

Micrometrical Measures of Double Stars, made with the 10*-in. Refractor of the Armagh Observatory.* By Mervyn A. Ellison.

(*Communicated by the Director, Armagh Observatory.*)

Star.	1925. R.A.	Dec.	P.	D.	Mags.	No. of Nights.	Date 1920. +	
	h m	° ′	°	″				
Σ 3061	0 1·9	+17 25	145·7	7·59	8·0, 8·0	2	6·006	
Σ 3060	0 2·0	+17 40	123·0	3·58	8·5, 8·5	2	6·006	
Σ 67	0 48·1	+10 12	358·3	1·99	8·3, 9·0	2	6·006	
OΣ 98 (14 Orionis)	5 3·8	+ 8 23	129·1	0·74	6·0, 6·8	3	6·080	
Σ 795 (52 Orionis)	5 44·0	+ 6 26	209·3	1·51	6·2, 6·2	3	6·058	
Σ 1536 (ι Leonis)	9 20·0	+10 58	29·3	1·50	3·9, 7·1	4	6·235	
Σ 1356 (ω Leonis)	9 24·5	+ 9 24	132·1	0·89	6·2, 7·0	3	6·205	
Σ 1454	10 34·0	+27 0	320·5	2·48	8·0, 10·0	2	6·298	
β 913 (40 Leo. Min.)	10 38·9	+26 43	116·0	15·26	6·0, 12·9	2	6·321	A.B.
Σ 1785	13 45·7	+27 22	47·0	1·14	7·2, 7·5	3	6·242	
Σ 1956	15 30·6	+42 14	37·6	1·53	6·2, 8·9	3	5·584	
Σ 2022	16 9·7	+26 51	321·7	2·41	6·2, 9·8	2	5·561	
Σ 2140 (α Herculis)	17 11·2	+14 29	111·6	4·57	3·0, 6·1	4	5·561	
Sch. 541	17 15·0	+32 34	290·1	4·75	9·7, 10·3	2	6·671	
β 630	17 16·4	+32 25	221·7	1·60	9·3, 11·0	3	6·684	
Sch. 543	17 33·7	+32 46	178·7	7·27	9·2, 11·0	2	6·677	
Sch. 544	17 34·4	+31 27	278·3	9·75	9·7, 11·3	2	6·677	
Sch. 552	17 52·9	+31 18	233·4	5·57	10·1, 10·6	2	6·677	
Sch. 600	19 23·3	+35 46	79·8	7·85	9·5, 10·3	2	6·671	A.B.
β 55	19 42·7	+10 22	24·2	4·13	9·6, 9·7	2	5·623	A.B.
			257·7	36·53	9·6	2	5·623	A.C.
Espin 200	19 52·4	+34 23	229·0	4·39	10·0, 10·0	3	5·877	
Σ 2824 (κ Pegasi)	21 41·2	+25 18	296·7	12·61	3·9, 10·8	3	5·786	
Σ 2934	22 38·2	+21 2	112·7	0·84	8·2, 9·2	4	5·835	

Notes.

β 913. B first seen in bright moonlight. C and D not seen.
Sch. 600. B.D.+35°·3613. B.D. numbers of Sch. 600 and 601 have been changed about in Potsdam Catalogue. Espin measured A.C. 1926·645, P. 99°·6, D. 6″·91, 2 nts., C=12·5. The apparent P.M. of B is 0″·21 at P. 28°·8. Hertzsprung finds the motion in A and says C is moving with it, while B is fixed. For A he gives P.M. 0″·17 in 199°.
Σ 2824 ($\varkappa$ Pegasi). A number of minute pairs and two triples in this field.
Espin 200. Holmes 32, also Scheiner 727.
Sch. 552. Also Espin 342.

New Double Stars Found and Measured in the Course of Observation.

No.	1925. R.A.	Dec.	P.	D.	Mags.	No. of Nights.	Date 1920. +
	h m	° ′	°	″			
1	10 59·1	+22 11	354·0	10·29	10·7, 10·9	2	6·268
2	11 8·8	+22 23	87·1	3·55	8·7, 9·7	2	6·407
3	11 10·7	+23 22	132·5	8·98	10·4, 10·6	2	6·294
4	19 13·2	+28 19	316·0	4·33	11·3, 11·5	3	5·705
5	19 14·0	+27 4	112·1	9·46	10·8, 11·0	2	5·732
6	23 29·1	+21 43	1·9	6·14	11·1, 11·3	2	5·739

Notes.

2. B.D. + 22°·2331. Fine pair.
3. 10′ *sp*, 72 Leonis.
4. 10½′ *n*, Σ 2491.
5. 15′ *sf*, OΣ 371.
6. 25′ *s*, 16′ *p*, 71 Pegasi. Holmes' triple 6′ *nf*.

1927 *February* 7.

Occultations observed at Herne in 1926. By B. M. Peek, M.A.

Co-ordinates of Observatory.
Longitude $0^h\ 4^m\ 27^s{\cdot}74$ E.
Latitude 51° 21′ 17″·8 N.
Altitude 47 ft.
Log $\rho \sin \phi'$ 9·8906302.
Log $\rho \cos \phi'$ 9·7964206.

Date. 1926	Star.	Mag.	Phenomenon.	G.M.T. h m s
Feb. 25	12 B. Leonis	6·3	D	21 25 20·2
May 17	B.D.+20° 2132	8·7	D	21 19 0·7
17	W.Z.C. 584	6·7	D	21 24 41·9
17	W.Z.C. 586	8·6	D	21 57 14·3
17	W.Z.C. 588	6·6	D	22 17 40·8
17	B.D.+20° 2153	8·0	D	22 22 35·5
17	B.D.+19° 2073	8·0	D	22 36 43·6
Sept. 18	W.Z.C. 1409	6·8	D	20 19 3·0
19	W.Z.C. 1487	6·6	D	22 31 47·1
Dec. 19	μ Geminorum	3·2	R	19 31 9·7

All of the above were observed at the dark limb of the moon. The times have been corrected by $0^s{\cdot}5$, which is the experimentally determined lag of the observer in starting the stop-watch. Any occultation which is known to have been timed later than normal has been further corrected by an estimated fraction of a second.

The 6-inch Zeiss refractor was used throughout, except on February 25, when the observation was made with the 5-inch refractor on the same mounting, the tube for the Zeiss not having been completed.

Occultations of Stars by the Moon, observed at Tomsk University Observatory during the years 1925, 1926. By Professor N. Goryatschew.

(*Communicated by the Secretaries.*)

($\phi = 56° 28' 07''$; $\lambda = 5^h 39^m 47^s{\cdot}4$ east of Greenwich.)

Date.	Star.	Magn.	Phase.	Local Sidereal Time.	Observer.
1925.				h m s	
VIII. 2	*o* Sagittarii	3·9	D	18 05 03·3	S.
7	ψ^2 Aquarii	4·6	R	22 20 08·2	S.
IX. 7	ξ^2 Ceti	4·3	D	0 06 09·0	Grt.
7	ξ^2 Ceti	4·3	R	1 14 40·8	Grt.
11	B.D.+19°·1110	6·0	R	1 10 21·3	Grt.
11	57 Orionis	5·8	R	2 50 52·4	Grt.
X. 9	16 Geminorum	6·2	R	4 15 37·1	*
XI. 25	64 B Ceti	7·0	D	23 29 57·6	*
XII. 2	χ^1 Orionis	4·5	R	10 27 49·0	*
3	56 Geminorum	5·2	R	0 44 48·6	Grt.
24	311 B Piscium	7·1	D	2 35 50·69	*
1926.					
III. 24	176 B Cancri	6·6	D	8 57 55·2	R.
VII. 21	131 B Scorpii	5·5	D	17 12 12·7	*
30	20 Ceti	4·9	R	23 38 28·6	*
30	20 Ceti	4·9	R	23 38 29·1	K.
VIII. 31	191 B Orionis	7·4	R	22 09 32·2	*
IX. 2	*d* Geminorum	5·2	R	2 51 18·9	*
4	35 Cancri	6·6	R	2 36 38·4	*
15	B.D.−22°·4436	7·1	D	19 12 20·0	*
X. 28	μ Cancri	5·5	R	7 25 04·5	*
28	μ Cancri	5·5	R	7 25 03·8	K.
XI. 15	30 Piscium	4·7	D	22 50 09·8	*
15	30 Piscium	4·7	D	22 50 09·9	K.
15	33 Piscium	4·7	D	0 32 47·5	*
15	33 Piscium	4·7	D	0 32 47·8	K.

Observers.—S. = Stud. Stzirshetrky; Grt. = Mrs. Gort-de-Grott; R. = Mrs. Riasanova; K. = Mrs. Kisseleva; * = Professor N. Goryatschew.

Instrument.—Zeiss telescope, 90 mm. aperture; magnifying power from 90 to 150. Observations made with eye and ear.

Apparent Places of Occulted Stars from W.Z.C. (Hedrick).

N.W.Z.C.	Star.	Magn.	α.	δ.
			h m s	° ′ ″
25	64 B Ceti	7·0	0 25 51·08	− 3 14 59·1
118	311 B Piscium	7·1	1 56 05·30	+ 5 40 30·7
616	176 B Cancri	6·6	8 55 00·73	+18 25 22·6
394	191 B Orionis	7·4	5 43 14·17	+20 54 55·1
582	35 Cancri	6·6	8 31 04·39	+19 50 42·3
152	B.D. −22°·4436	7·1	17 46 39·55	−22 53 56·9

Tomsk, Siberia, U.S.S.R.,
University Astronomical Observatory:
1927 *January* 23.

Erratum.

On p. 335, lines 15 and 14 from the bottom, for the parenthetical clause—"based on . . . Coblentz," *read*: "which he finds consistent with the density distribution of the major planets as deduced from their polar flattening, and is seemingly supported by determinations of their radiation by Menzel, Coblentz, and Lampland (*Ap. J.*, lxiii. 177 *et seq.*)."